A Model for Predicting Educational Enrolment and Output in the Post-secondary Educational System of Norway

by

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A MODEL FOR PREDICTING EDUCATIONAL ENROLMENT
AND OUTPUT IN THE POST-SECONDARY
EDUCATIONAL SYSTEM OF NORWAY

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Foreword

This report describes an attempt to develop a mathematical model designed for the purpose of forecasting enrolment, graduation and drop-out in universities and other institutions engaged in post-secondary education.

A few numerical examples have already been calculated by the model. These examples are based on different assumptions concerning future participation rates in secondary school, and transition from secondary school to universities. The main results of the calculations are presented in tables in the report.

Mrs. Eva Birkeland, consultant at the Research Department, has conducted the theoretical work on the model and prepared this report. The research work on mathematical models of educational systems done by Assistant Professor Thonstad has been of special value in this work, as well as empirical studies on student behaviour, undertaken by other members of the Research Department. Mr. Eivind Hoffmann has been engaged in compiling and preparing data for the model, and the model has been adapted to Electronic Data Processing in cooperation with Mr. Tor Kottmann at the Norwegian Computing Centre.

Sigmund Vangsnes

Department Director

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I. INTRODUCTION.

Institute for Studies in Research and Higher Education, The Norwegian Research Council for Science and the Humanities, has for several years been preparing predictions of enrolment in institutions of higher learning as well as forecasts of supply of and demand for qualified manpower. These analyses are of great interest to both occupational and educational planning programmes. They treat the specific fields separately. Thus the danger of making a contorted over-all picture becomes involved because the assumptions on which the studies have been based, may lack the necessary correspondence. With this point in mind, the Research Department in 1965 started to work on a model for the post-secondary educational system in Norway. For a start, we made use of the educational prediction model developed by Assistant Professor Tore Thonstad, and we have attempted to develop a model that has validity especially for predicting educational enrolment and output in post-secondary education.

In principle, the model covers all current education possibilities based on gymnas graduation, both academic and non-academic \(^1\). A total of 58 educational processes is for the time being included in the model: 9 basic courses in non-restricted fields of study, 10 advanced courses in non-restricted fields of study, 18 restricted fields of study, 5 fields of study in foreign countries, and 16 non-academic educational processes.

The model has not been constructed with reference to sex differences in educational behaviour, but is applicable to either or both sexes.

The prediction period was decided to be twenty years, but we are free to make it as long as wanted.

The object of the model is to serve as an aid for educational planning.

The model was adapted to EDP (Electronic Data Processing), and the computing programme was made at the Norwegian Computing Centre.

The model was constructed with available data taken into consideration, but we do not have as reliable information as desirable, nor do we have a workable theory of study patterns. The model can therefore only give forecasts based on rather uncertain information regarding the number of gymnas graduates, propensities for certain fields of study, the interval between matriculation examination and commencement of study, enrolment restrictions, and study patterns. None of these can be expected to remain constant over

\(^1\) The gymnas is a secondary school with a final examination which one has to pass to be entitled to enter university.
a long period. In the model, therefore, all coefficients may, in principle, vary with time.

II. DESCRIPTION OF THE MODEL.

Each educational process consists of ten activities. The first activity applies to students in their first year of study, the second to those in their second year, and so forth. The study period is considered as a whole, that is, without regarding possible interruptions.

We begin by establishing the number of students to be found in each activity at a certain time. How many of these students will, in the course of a year, graduate from each process, how many will drop out from each process, how many will transfer from one process to another, and how many will be in the various activities the following year?

Admission to restricted fields of study, to non-academic education and to foreign study is included as data, whereas admission to non-restricted fields of study is calculated from the model. Primary admission to non-restricted fields of study refers to the first admission of gymnas graduates. It is made up partly of new graduates, partly of graduates from previous years. An interval of up to three years between gymnas graduation and commencement of study is employed in the model. This is also the case for restricted fields of study, foreign study, and non-academic education.

Secondary admission to non-restricted fields of study (which can be either a positive or a negative figure) covers the students who are redistributed from restricted fields of study or foreign study caused by limited capacity and altered admission policies. The third form of considered admission to non-restricted fields of study is transition from other non-restricted fields of study.

To compute primary admission to non-restricted fields of study, we require information on the size of the graduating gymnas cohorts, \( A(t+T) \). (Explanation of the symbols can be found in Appendix 1). We assume that for every gymnas cohort there are certain propensities for certain fields of study, \( a_{r}(t+T) \), i.e., a definite fraction of the cohort wishes to
enter a certain field of study. The quantity, \( A_{r}(t+T) \), is derived as the product of gymnas cohort size and the study propensity:

\[
A_{r}(t+T) = a_{r}(t+T) \cdot A(t+T)
\]

A certain fraction of these begin their studies in the same year as graduation from the gymnas, \( k_{0r}(t+T) \), some begin the following year, \( k_{1r}(t+T) \), some two years later, \( k_{2r}(t+T) \), and the rest begins three years later, \( k_{3r}(t+T) \). These fractions are designated here as interval coefficients. The number of graduates from one year's cohort entering a definite field of study in the same year, \( Y_{r}(t+T) \), the year after, \( Z_{r}(t+T) \), two years after, \( W_{r}(t+T) \), and three years after, \( U_{r}(t+T) \), respectively, are calculated as the product of the relevant interval coefficient and the number of students wanting to enter a particular field of study.

\[
\begin{align*}
Y_{r}(t+T) &= k_{0r}(t+T) \cdot A_{r}(t+T) \\
Z_{r}(t+T) &= k_{1r}(t+T) \cdot A_{r}(t+T) \\
W_{r}(t+T) &= k_{2r}(t+T) \cdot A_{r}(t+T) \\
U_{r}(t+T) &= k_{3r}(t+T) \cdot A_{r}(t+T)
\end{align*}
\]

The same method is used to compute the number of gymnas graduates from one year's cohort entering a restricted field of study or a field of study in a foreign country in the same year, the year after, \( X_{r}(t+T) \), two years after, \( W_{r}(t+T) \), and three years after, \( U_{r}(t+T) \), respectively, are calculated as the product of the relevant interval coefficient and admission for the pertinent year, \( X_{r}(t+T) \):

\[
\begin{align*}
Y_{r}(t+T) &= l_{0r}(t+T) \cdot X_{r}(t+T) \\
Z_{r}(t+T) &= l_{1r}(t+T+1) \cdot X_{r}(t+T+1) \\
W_{r}(t+T) &= l_{2r}(t+T+2) \cdot X_{r}(t+T+2) \\
U_{r}(t+T) &= l_{3r}(t+T+3) \cdot X_{r}(t+T+3)
\end{align*}
\]

Subsequent to the above calculations, one can compute the number of gymnas graduates from each year who will sooner or later enter the field of study of their first choice, \( B(t+T) \):

\[
B(t+T) = \sum_{r=1}^{m} \left( Y_{r}(t+T) + Z_{r}(t+T) + W_{r}(t+T) + U_{r}(t+T) \right)
\]

Assuming that for each gymnas cohort there is a certain total study propensity, \( s(t+T) \), we can calculate the number of graduates wanting further academic education, \( S(t+T) \):

\[
S(t+T) = s(t+T) \cdot A(t+T)
\]
If \( S(t+T) = B(t+T) \) there is a correspondence between the presumed total study propensity and the proportion of gymnas graduates being admitted to the field of study of their first choice. In the opposite case, we obtain a difference that can be either positive or negative:

\[
R(t+T) = S(t+T) - B(t+T)
\]

If \( R(t+T) \) is positive, we have a number of gymnas graduates not being admitted to the field of study of their first choice. These students must either refrain from further academic education, or places must be made available for them in the restricted fields of study, or they must be distributed among the non-restricted fields of study. This distribution is made with the aid of distribution coefficients, \( \gamma_r(t+T) \) and we call this secondary admission.\(^1\)

The new admissions, \( C_r(t+T) \), to the basic courses in non-restricted fields of study are then:

\[
C_r(t+T) = Y_r(t+T) + Z_r(t+T-1) + W_r(t+T-2) + U_r(t+T-3) + \gamma_r(t+T) R(t+T)
\]

of which all but the last quantities represent the primary admission from the last four gymnas cohorts, and the last quantity represents the secondary admissions.

If \( R(t+T) \) is negative, the calculated number of entrants to academic educational processes is larger than the number that according to our assumed total study propensity, wants further academic education. On one hand, we may accept this. If so, we accept, as a corrected estimate for the total study propensity, the proportion which is consistent with the calculated number. On the other hand, we may maintain our first estimate by reducing the number of entrants to the basic courses in the non-restricted fields of study. Thereby we assume that all the available places in the restricted fields of study are utilized.

In addition to the new admissions, transition from other fields of study may occur. This is calculated as a fraction, \( m_{rs}(t+T) \), of drop-outs from other fields of study, \( M_s(t+T) \).

Admission to an advanced course in a non-restricted field of study is determined as a fraction, \( e_{rs}(t+T) \), of those who have graduated from the basic courses the previous year, \( E_s(t+T) \). A possible interval between the basic course and the advanced course is included in the total study period for the advanced course. The formula for calculation of admission to the

\(^1\) For the secondary admission, an interval is not considered.
non-restricted fields of study is:

$$X_r(t+T) = C_r(t+T) + \sum_{s=1}^{p} m_{r,s}(t+T) \cdot M_g(t+T) + \sum_{s=1}^{q} e_{r,s}(t+T) \cdot E_g(t+T),$$

of which the first two quantities refer to the basic courses, and the last refers to the advanced courses.

As mentioned above, admissions to the restricted fields of study and foreign study are included as data. The number of students in the first activity in each educational process is equal to total admissions to the process.

$$N_{r,1}(t+T) = X_r(t+T)$$

The number of students in the other activities is considered as a definite fraction, $c_{r,i-1}(t+T-1)$, of the number of students in the preceding activity of the previous year, $N_{r,i-1}(t+T-1)$:

$$N_{r,i}(t+T) = c_{r,i-1}(t+T-1) \cdot N_{r,i-1}(t+T-1) \quad (i = 2 \ldots 10)$$

The number of graduates from each activity is calculated as a definite fraction, $h_{r,i}(t+T-1)$ for graduation in autumn, and $v_{r,i}(t+T-1)$, for graduation in spring, of the total number of students who were enrolled in the activity at the beginning of the year, $N_{r,i}(t+T-1)$.

Thus the total number of graduates from an educational process during an academic year, $E_r(t+T)$, is the sum of the number of graduates from each activity.

$$E_r(t+T) = \sum_{i=1}^{10} (h_{r,i}(t+T-1) + v_{r,i}(t+T-1)) \cdot N_{r,i}(t+T-1)$$

The number of drop-outs from the various activities of an educational process during the academic year is calculated as a definite fraction, $f_{r,i}(t+T-1)$, of those enrolled at the beginning of the year. The total number of drop-outs from an educational process during one year is the sum of drop-outs from each activity.

$$M_r(t+T) = \sum_{i=1}^{10} f_{r,i}(t+T-1) \cdot N_{r,i}(t+T-1)$$
ON DATA AND RESULTS.

The following magnitudes can be calculated with the aid of the model:

1. Admission to the courses in non-restricted fields of study for each year of the prediction period, \( X_r(t+T) \).

2. The total number of students in each activity for each year of the prediction period, \( N_{r,i}(t+T) \).

3. The number of graduated candidates from each educational process for each year of the prediction period \( E_r(t+T) \).

4. The total number of drop-outs from each educational process for each year of the prediction period, \( M_r(t+T) \).

The model requires the following data:

A. The number of students in each activity in the basis year, \( N_{r,i}(t) \).

B. Data concerning the transition from gymnasium to the educational processes.

1. The total number of gymnasium graduates for each year in the prediction period \( A(t+T) \). Here we can either use already prepared predictions, or calculate an estimate of graduates as a subprocess of the model.

2. The number of admissions to restricted fields of study, to foreign study and to non-academic education for every year in the prediction period, \( X_r(t+T) \).

3. Total study propensity for each graduating cohort from the gymnasium, \( s(t+T) \).

4. Particular propensities for basic courses in the non-restricted fields of study for each graduating gymnas cohort, \( a_r(t+T) \).

5. The coefficients for interval between gymnas graduation and commencement of each educational process, \( k_{j,r}(t+T) \) and \( l_{j,r}(t+T) \).

6. Distribution coefficients for secondary admission to basic courses in non-restricted fields of study.

C. Data concerning study patterns.

1. The coefficients for transition between the non-restricted fields of study for each year in the prediction period, \( m_{r,s}(t+T) \).

2. The coefficients for advanced-level education (transition from a basic course to an advanced course in non-restricted fields of study) for each year in the prediction period, \( e_{r,s}(t+T) \).
3. Continuation coefficients (transition from one activity to the next in the same educational process) for each year of the prediction period, \( c_{r,i}(t+T) \).

4. Graduation coefficients (graduation in respectively autumn and spring from each activity) for each year in the prediction period, \( h_{r,i}(t+T) \) and \( v_{r,i}(t+T) \).

5. Drop-out coefficients (drop-out from each activity) for each year of the period, \( f_{r,i}(t+T) \).

We have made five numerical examples based on different data concerning the transition from gymnas to the educational processes, while the number of students in each activity in the basis year and the data concerning study patterns are the same in all examples.

The figures for admission to restricted fields of study, to foreign study and to non-academic education were determined on the basis of all available information about present and future enrolment capacities. The admission figures for these educational processes are the same in all the numerical examples. The estimates concerning restricted fields of study, foreign study, and non-academic education will consequently be the same in all the numerical examples.

The basis year for the calculations is 1964, and the prediction period is 1965 - 1985. The number of students and their distribution among the activities in the basis year were determined chiefly on the basis of statistical material on students from the Central Bureau of Statistics.

In order to establish the estimates for the coefficients for the interval between gymnas graduation and commencement of university study, for continuation, for drop-out, and for graduation from the educational processes, we examined the study patterns of the gymnas cohorts of 1951 and 1958. For most of the processes we also collected supplementary information.

On the basis of this information we drew a study profile for each educational process, i.e., a diagram which shows a drop-out curve and a graduation curve.
On the basis of these study profiles, we determined the coefficients for graduation, continuation, and drop-out.

Since we have relatively little information about transition between the processes, these coefficients are all determined to be zero in these examples.

Special information was gathered in order to establish the coefficients for advanced-level education.

We use two alternative methods for calculating admissions to the basic courses in non-restricted fields of study. In alternative I, the calculations are made from the assumed particular study propensities. In alternative II, we also have secondary admissions because of "bottlenecks" in the system. Lacking information about the redistribution, we calculate with secondary admissions only to social and natural sciences. The distribution coefficients were decided to be proportional to the primary admission to these studies.
The first numerical example.

The size of the gymnas cohorts 1966-1985 was estimated from the prediction prepared by The Norwegian Research Council for Science and the Humanities, alternative II (see appendix 3) plus a supplement of 10 percent, representing graduates from the commercial gymnas. The total study propensity was assumed to be 0.50 each year of the period.

The particular study propensities for the basic courses in non-restricted fields of study were estimated on the basis of data from 1965 and were assumed to be the same for each gymnas cohort in the prediction period:

<table>
<thead>
<tr>
<th>Field of Study</th>
<th>Propensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Humanities</td>
<td>16.40</td>
</tr>
<tr>
<td>Law</td>
<td>4.00</td>
</tr>
<tr>
<td>Theology</td>
<td>0.90</td>
</tr>
<tr>
<td>Natural sciences</td>
<td>9.00</td>
</tr>
<tr>
<td>Economics</td>
<td>2.00</td>
</tr>
<tr>
<td>Psychology</td>
<td>2.00</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>0.80</td>
</tr>
<tr>
<td>Sociology</td>
<td>0.30</td>
</tr>
<tr>
<td>Political science</td>
<td>0.50</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>36.00</strong></td>
</tr>
</tbody>
</table>

Some main results are shown in Table 1 and Table 2.
Table 1. Number of students. First numerical example.

<table>
<thead>
<tr>
<th>Year</th>
<th>Humanities</th>
<th>Law</th>
<th>Theology</th>
<th>Natural sciences</th>
<th>Economics</th>
<th>Psychology</th>
<th>Pedagogy</th>
<th>Sociology</th>
<th>Political science</th>
<th>Number of students in non-restricted fields of study</th>
<th>Number of students in restricted fields of study</th>
<th>Total number of students in academic fields of study in Norway</th>
<th>Number of students in academic fields of study in foreign countries</th>
<th>Number of students in non-academic education</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>5293</td>
<td>1190</td>
<td>504</td>
<td>3791</td>
<td>437</td>
<td>516</td>
<td>182</td>
<td>54</td>
<td>159</td>
<td>12126</td>
<td>4691</td>
<td>16817</td>
<td>3659</td>
<td>6068</td>
</tr>
<tr>
<td>1970</td>
<td>9472</td>
<td>2704</td>
<td>701</td>
<td>702</td>
<td>1338</td>
<td>1037</td>
<td>291</td>
<td>168</td>
<td>360</td>
<td>23082</td>
<td>4208</td>
<td>30539</td>
<td>197</td>
<td>10428</td>
</tr>
<tr>
<td>1975</td>
<td>9278</td>
<td>2704</td>
<td>701</td>
<td>5812</td>
<td>1073</td>
<td>1258</td>
<td>230</td>
<td>136</td>
<td>295</td>
<td>21089</td>
<td>4844</td>
<td>28546</td>
<td>586</td>
<td>10453</td>
</tr>
<tr>
<td>1980</td>
<td>12152</td>
<td>3499</td>
<td>859</td>
<td>8919</td>
<td>1712</td>
<td>1126</td>
<td>360</td>
<td>192</td>
<td>484</td>
<td>29458</td>
<td>578</td>
<td>23089</td>
<td>829</td>
<td>10536</td>
</tr>
<tr>
<td>1985</td>
<td>11984</td>
<td>3499</td>
<td>859</td>
<td>7812</td>
<td>1518</td>
<td>1494</td>
<td>331</td>
<td>256</td>
<td>427</td>
<td>27748</td>
<td>566</td>
<td>21089</td>
<td>1026</td>
<td>10534</td>
</tr>
</tbody>
</table>
Table 2. Average number of graduates per year from academic fields of study. First numerical example.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Humanities</td>
<td>125</td>
<td>125</td>
<td>389</td>
<td>478</td>
</tr>
<tr>
<td>Law</td>
<td>122</td>
<td>287</td>
<td>382</td>
<td>463</td>
</tr>
<tr>
<td>Theology</td>
<td>41</td>
<td>61</td>
<td>73</td>
<td>89</td>
</tr>
<tr>
<td>Natural sciences</td>
<td>227</td>
<td>482</td>
<td>638</td>
<td>779</td>
</tr>
<tr>
<td>Economics</td>
<td>38</td>
<td>134</td>
<td>176</td>
<td>214</td>
</tr>
<tr>
<td>Psychology</td>
<td>41</td>
<td>94</td>
<td>112</td>
<td>136</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>8</td>
<td>15</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Sociology</td>
<td>5</td>
<td>13</td>
<td>21</td>
<td>26</td>
</tr>
<tr>
<td>Political science</td>
<td>6</td>
<td>29</td>
<td>43</td>
<td>52</td>
</tr>
<tr>
<td>Medicine in Norway</td>
<td>141</td>
<td>134</td>
<td>389</td>
<td>478</td>
</tr>
<tr>
<td>Medicine in foreign countries</td>
<td>116</td>
<td>145</td>
<td>149</td>
<td>159</td>
</tr>
<tr>
<td>Dentistry in Norway</td>
<td>46</td>
<td>149</td>
<td>164</td>
<td></td>
</tr>
<tr>
<td>Dentistry in foreign countries</td>
<td>79</td>
<td>58</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>Pharmacy</td>
<td>26</td>
<td>33</td>
<td>35</td>
<td>63</td>
</tr>
<tr>
<td>Veterinary medicine</td>
<td>25</td>
<td>33</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Agriculture</td>
<td>122</td>
<td>149</td>
<td>151</td>
<td>151</td>
</tr>
<tr>
<td>Business administration and economics in Norway</td>
<td>145</td>
<td>190</td>
<td>190</td>
<td>190</td>
</tr>
<tr>
<td>Business administration and economics in foreign countries</td>
<td>83</td>
<td>95</td>
<td>104</td>
<td>104</td>
</tr>
<tr>
<td>Engineering in Norway</td>
<td>407</td>
<td>645</td>
<td>708</td>
<td>799</td>
</tr>
<tr>
<td>Engineering in foreign countries</td>
<td>251</td>
<td>204</td>
<td>171</td>
<td>169</td>
</tr>
<tr>
<td>Architecture in Norway</td>
<td>54</td>
<td>75</td>
<td>89</td>
<td>89</td>
</tr>
<tr>
<td>Architecture in foreign countries</td>
<td>19</td>
<td>21</td>
<td>14</td>
<td>13</td>
</tr>
</tbody>
</table>
The second numerical example.

We used the same estimates for the size of the gymnas cohorts as in the first numerical example.

The total study propensity was assumed to be the same as in the first numerical example.

The particular propensities for the social sciences were assumed to increase from 1966-1985, while the particular propensity for humanities was assumed to decrease correspondingly.

The particular propensity for:

- Economics increases from 2.00 in 1966 to 3.00 in 1975
- Psychology  "  " 2.00  "  " 3.00  "  "  
- Sociology  "  " 0.30  "  " 2.00  "  "  
- Political science  " 0.60  "  " 2.00  "  "  
- Pedagogy  " 0.80  "  " 2.00  "  "  
- Humanities decreases  " 16.40  "  " 10.10  "  "  

From 1975 to 1985 the propensities were assumed to be as in 1975.

The other particular propensities were assumed to be as in the first numerical example.

Some main results are shown in Table 3 and Table 4.
Table 3. Number of students. Second numerical example.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Humanities</td>
<td>5293</td>
<td>8934</td>
<td>8786</td>
<td>9788</td>
<td>9651</td>
</tr>
<tr>
<td>Law</td>
<td>1190</td>
<td>2704</td>
<td>2704</td>
<td>3499</td>
<td>3499</td>
</tr>
<tr>
<td>Theology</td>
<td>504</td>
<td>710</td>
<td>710</td>
<td>859</td>
<td>859</td>
</tr>
<tr>
<td>Natural sciences</td>
<td>3791</td>
<td>6958</td>
<td>5830</td>
<td>8882</td>
<td>7906</td>
</tr>
<tr>
<td>Economics</td>
<td>437</td>
<td>1424</td>
<td>1146</td>
<td>2180</td>
<td>1970</td>
</tr>
<tr>
<td>Psychology</td>
<td>516</td>
<td>1186</td>
<td>980</td>
<td>1718</td>
<td>1566</td>
</tr>
<tr>
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<td>9960</td>
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<td>10453</td>
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<td>10534</td>
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</tbody>
</table>

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education is the same as in the first numerical example.
Table 4. Average number of graduates per year from academic fields of study. Second numerical example.

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<td>61</td>
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<td>Political science</td>
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<td>6</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.
The third numerical example.

The size of the gymnas cohorts 1966-1985 was estimated from the prediction prepared by the Norwegian Research Council for Science and the Humanities, alternative I (see appendix 3), plus a supplement of 10 percent representing graduates from the commercial gymnas.

The total study propensity was assumed to increase from 0.50 in 1966 to 0.60 in 1975 and to be 0.60 from 1975-1985. The particular study propensities were assumed to be the same as in the first numerical example.

Some main results are shown in Table 5 and Table 6.
Table 5. Number of students. Third numerical example.

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Number of students in restricted fields of study:

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<td>Alt.I</td>
<td>Alt.II</td>
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</table>

Number of students in academic fields of study in foreign countries:

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<td>10534</td>
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</table>

1) The number of students in restricted fields of study, in academic fields of study in foreign countries and in non-academic education are the same as in the first numerical example.
Table 6. Average number of graduates per year from academic fields of study. Third numerical example.

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</table>

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.
The fourth numerical example.

The size of the gymnas cohorts 1966–1985 was assumed to be the same as in the third numerical example. The study propensities (both total and particular) were assumed to be the same as in the first numerical example. Some main results are shown in Table 7 and Table 8.
Table 7. Number of students. Fourth numerical example.

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</table>

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education are the same as in the first numerical example.
Table 8. Average number of graduates per year from academic fields of study. Fourth numerical example.

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</table>

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.
The fifth numerical example.

The size of the gymnas cohorts 1966-1973 was assumed to be the same as in the first numerical example. The size of the gymnas cohorts 1974-1985 was assumed to be 25 percent of the $19^{1/2}$ years old persons plus a supplement of 10 percent representing graduates from the commercial gymnas. The study propensities (both total and particular) were assumed to be the same as in the first numerical example. Some main results are shown in Table 9 and Table 10.
Table 9. Number of students. Fifth numerical example.

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</table>

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education are the same as in the first numerical example.
Table 10. Average number of graduates per year from academic fields of study. Fifth numerical example.

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<td>5</td>
<td>16</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Political Science</td>
<td>6</td>
<td>29</td>
<td>43</td>
<td>49</td>
</tr>
</tbody>
</table>

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.
Appendix 1. A survey of symbols used in the model.

- \( r = s = 1 \) .... 58 educational processes.
- \( i = 1 \) .... 10 activities.
- \( j = 0 \) .... 3 years between gymnas graduation and commencement of study.
- \( t = \) basis year.
- \( T = 1 \) .... 21 years in the prediction period.

\[ A(t+T) \]
= The number of gymnas graduates in the year \( t+T \).

\[ A_r(t+T) \]
= The number of gymnas graduates in the year \( t+T \) wanting to enter a certain educational process.

\[ a_r(t+T) \]
= Propensity for a certain educational process for the \( t+T \)-cohort.

\[ Y_r(t+T) \]
= The number of gymnas graduates from the \( t+T \)-cohort entering a certain educational process the same year.

\[ Z_r(t+T) \]
= The number of gymnas graduates from the \( t+T \)-cohort entering a certain educational process the following year.

\[ W_r(t+T) \]
= The number of gymnas graduates from the \( t+T \)-cohort entering a certain educational process two years after.

\[ U_r(t+T) \]
= The number of gymnas graduates from the \( t+T \)-cohort entering a certain educational process three years after.

\[ k_{j,t}(t+T) \] and \[ l_{j,t}(t+T) \]
= The fraction of the \( t+T \)-cohort entering a certain educational process \( j \) years after.

\[ X_r(t+T) \]
= Admissions to a certain educational process in year \( t+T \).

\[ B(t+T) \]
= The total number of gymnas graduates from the \( t+T \)-cohort entering the field of study of their first choice.

\[ S(t+T) \]
= The number of gymnas graduates in the year \( t+T \) wanting further academic education.

\[ s(t+T) \]
= Total study propensity for the gymnas graduates in the year \( t+T \).

\[ R(t+T) \]
= The difference between \( S(t+T) \) and \( B(t+T) \).

\[ C_r(t+T) \]
= New admissions to a certain educational process in the year \( t+T \).

\[ Y_r(t+T) \]
= Distribution key for secondary admissions to basic courses in non-restricted fields of study in the year \( t+T \).

\[ N_{r,i}(t+T) \]
= The number of students in activity No. \( r,i \) in the year \( t+T \).

\[ M_r(t+T) \]
= The number of drop-outs from a certain educational process during the academic year \( t+T-1/t+T \).

\[ f_{r,i}(t+T) \]
= Transition coefficient for drop-out.
$E_r(t/T) = \text{The number of graduates from a certain educational process during the academic year } t+T-1/t+T.$

$h_{r,i}(t+T) = \text{The fraction of the number of students enrolled in a certain activity at the beginning of the year graduating during the academic year } t+T/t+T+1. \ (h_{r,i}(t+T) \text{ for graduation in autumn and } v_{r,i}(t+T) \text{ for graduation in spring}).$

$m_{r,s}(t+T) = \text{The fraction of the number of drop-outs from an educational process during the academic year } t+T-1/t+T \text{ entering a certain educational process in the year } t+T.$

$e_{r,s}(t+T) = \text{The fraction of the number of graduates from a basic course of study during the academic year } t+T-1/t+T \text{ entering an advanced course of study in the year } t+T.$

$c_{r,i}(t+T) = \text{The fraction of the number of students enrolled in activity No. } r,i \text{ in a certain educational process at the beginning of the year } t+T \text{ who next year will be enrolled in the activity } r,i+1.$

$N_r(t+T) = \text{The number of students enrolled in a certain educational process in the year } t+T.$

$n = \text{The number of fields of study.}$

$m = \text{The number of academic fields of study.}$

$p = \text{The number of non-restricted fields of study.}$

$q = \text{The number of basic courses in non-restricted fields of study.}$
Appendix 2. Model of the Post-gymnas Educational System.

1. \( A_r(t+T) = a_r(t+T) A(t+T) \)

2. \( Y_r(t+T) = k_{0,r}(t+T) A_r(t+T) \)
   \( Z_r(t+T) = k_{1,r}(t+T) A_r(t+T) \)
   \( W_r(t+T) = k_{2,r}(t+T) A_r(t+T) \)
   \( U_r(t+T) = k_{3,r}(t+T) A_r(t+T) \)

3. \( Y_r(t+T) = l_{0,r}(t+T) X_r(t+T) \)
   \( Z_r(t+T) = l_{1,r}(t+T+1) X_r(t+T+1) \)
   \( W_r(t+T) = l_{2,r}(t+T+2) X_r(t+T+2) \)
   \( U_r(t+T) = l_{3,r}(t+T+3) X_r(t+T+3) \)

4. \( B(t+T) = \sum_{r=1}^{M} (Y_r(t+T) + Z_r(t+T) + W_r(t+T) + U_r(t+T)) \)

5. \( S(t+T) = s(t+T) A(t+T) \)

6. \( R(t+T) = S(t+T) - B(t+T) \)

7. \( C_r(t+T) = Y_r(t+T) + Z_r(t+T-1) + W_r(t+T-2) + U_r(t+T-3) + Y_r(t+T) \cdot R(t+T) \)

8. \( M_r(t+T) = \sum_{i=1}^{10} f_{r,i}(t+T-1) N_{r,i}(t+T-1) \)

9. \( E_r(t+T) = \sum_{i=1}^{10} (h_{r,i}(t+T-1) + v_{r,i}(t+T-1))N_{r,i}(t+T-1) \)

10. \( X_r(t+T) = C_r(t+T) + \sum_{s=1}^{p} m_{r,s}(t+T) M_s(t+T) + \sum_{s=1}^{q} e_{r,s}(t+T) E_s(t+T) \)

11. \( N_{r,i}(t+T) = X_r(t+T) \) for \( i = 1 \)

12. \( N_{r,i}(t+T) = c_{r,i-1}(t+T-1) N_{r,i-1}(t+T-1) \) for \( i = 2 \ldots 10 \)

13. \( N_r(t+T) = \sum_{i=1}^{10} N_{r,i}(t+T) \)

14. \( N(t+T) = \sum_{r} N_r(t+T) \)
Appendix 3.

Estimates on pupil frequencies and gymnas graduate frequencies in the gymnas 1963/64 - 1984/85.

<table>
<thead>
<tr>
<th>Session</th>
<th>Pupil frequencies in first gymnas cohort</th>
<th>Gymnas graduate frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Alternative I</td>
<td>Alternative II</td>
</tr>
<tr>
<td>1963/64</td>
<td>19.3</td>
<td>23.9</td>
</tr>
<tr>
<td>1964/65</td>
<td>21.6</td>
<td>24.9</td>
</tr>
<tr>
<td>1965/66</td>
<td>22.9</td>
<td>25.9</td>
</tr>
<tr>
<td></td>
<td>25.7</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td>28.5</td>
<td>28.9</td>
</tr>
<tr>
<td>1967/68</td>
<td>31.3</td>
<td>30.9</td>
</tr>
<tr>
<td></td>
<td>32.7</td>
<td>31.9</td>
</tr>
<tr>
<td>1968/69</td>
<td>35.5</td>
<td>32.9</td>
</tr>
<tr>
<td></td>
<td>36.9</td>
<td>33.9</td>
</tr>
<tr>
<td>1969/70</td>
<td>39.7</td>
<td>34.9</td>
</tr>
<tr>
<td></td>
<td>41.1</td>
<td>35.9</td>
</tr>
<tr>
<td>1970/71</td>
<td>42.5</td>
<td>36.9</td>
</tr>
<tr>
<td></td>
<td>43.9</td>
<td>37.9</td>
</tr>
<tr>
<td>1971/72</td>
<td>45.3</td>
<td>38.9</td>
</tr>
<tr>
<td></td>
<td>46.7</td>
<td>39.9</td>
</tr>
<tr>
<td>1972/73</td>
<td>48.1</td>
<td>40.9</td>
</tr>
<tr>
<td></td>
<td>49.5</td>
<td>41.9</td>
</tr>
</tbody>
</table>

1) The number of gymnas graduates in the last three years has been about 90% of the number of pupils in the first gymnas cohort two years earlier. The increase in the pupil frequencies the last eight years has been 1.4 percent per year, on an average. In alternative I, this increase is assumed to continue. In alternative II, the assumption is an increase of 1.0 percent per year.

2) First class in the 3-year gymnas, or third class in the 5-year gymnas.

Prediction on the number of gymnas graduates, according to the above assumptions:

<table>
<thead>
<tr>
<th>Year</th>
<th>Alt. I</th>
<th>Alt. II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>11,536</td>
<td>11,536</td>
</tr>
<tr>
<td>1967</td>
<td>12,397</td>
<td>12,397</td>
</tr>
<tr>
<td>1968</td>
<td>12,690</td>
<td>12,690</td>
</tr>
<tr>
<td>1969</td>
<td>13,206</td>
<td>12,965</td>
</tr>
<tr>
<td>1970</td>
<td>13,745</td>
<td>13,328</td>
</tr>
<tr>
<td>1971</td>
<td>14,525</td>
<td>13,870</td>
</tr>
<tr>
<td>1972</td>
<td>15,595</td>
<td>14,685</td>
</tr>
<tr>
<td>1973</td>
<td>16,350</td>
<td>15,256</td>
</tr>
<tr>
<td>1974</td>
<td>17,165</td>
<td>15,826</td>
</tr>
<tr>
<td>1975</td>
<td>18,159</td>
<td>16,611</td>
</tr>
</tbody>
</table>