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A Model for Predicting Educational Enrolment and Output in the Post-secondary Educational System of Norway

by

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A MODEL FOR PREDICTING EDUCATIONAL ENROLMENT AND OUTPUT IN THE POST-SECONDARY EDUCATIONAL SYSTEM OF NORWAY

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Foreword

This report describes an attempt to develop a mathematical model designed for the purpose of forecasting enrolment, graduation and drop-out in universities and other institutions engaged in post-secondary education.

A few numerical examples have already been calculated by the model. These examples are based on different assumptions concerning future participation rates in secondary school, and transition from secondary school to universities. The main results of the calculations are presented in tables in the report.

Mrs. Eva Birkeland, consultant at the Research Department, has conducted the theoretical work on the model and prepared this report. The research work on mathematical models of educational systems done by Assistant Professor Thonstad has been of special value in this work, as well as empirical studies on student behaviour, undertaken by other members of the Research Department. Mr. Eivind Hoffmann has been engaged in compiling and preparing data for the model, and the model has been adapted to Electronic Data Processing in cooperation with Mr. Tor Kottmann at the Norwegian Computing Centre.

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C O N T E N T S .

I. INTRODUCTION.

Institute for Studies in Research and Higher Education, The Norwegian Research Council for Science and the Humanities, has for several years been preparing predictions of enrolment in institutions of higher learning as well as forecasts of supply of and demand for qualified manpower. These analyses are of great interest to both occupational and educational planning programmes. They treat the specific fields separately. Thus the danger of making a contorted over-all picture becomes involved because the assumptions on which the studies have been based, may lack the necessary correspondence. With this point in mind, the Research Department in 1965 started to work on a model for the post-secondary educational system in Norway. For a start, we made use of the educational prediction model developed by Assistant Professor Tore Thonstad, and we have attempted to develop a model that has validity especially for predicting educational enrolment and output in post-secondary education.

In principle, the model covers all current education possibilities based on gymnas graduation, both academic and non-academic [1]. A total of 58 educational processes is for the time being included in the model: 9 basic courses in non-restricted fields of study, 10 advanced courses in non-restricted fields of study, 18 restricted fields of study, 5 fields of study in foreign countries, and 16 non-academic educational processes.

The model has not been constructed with reference to sex differences in educational behaviour, but is applicable to either or both sexes.

The prediction period was decided to be twenty years, but we are free to make it as long as wanted.

The object of the model is to serve as an aid for educational planning.

The model was adapted to EDP (Electronic Data Processing), and the computing programme was made at the Norwegian Computing Centre.

The model was constructed with available data taken into consideration, but we do not have as reliable information as desirable, nor do we have a workable theory of study patterns. The model can therefore only give forecasts based on rather uncertain information regarding the number of gymnas graduates, propensities for certain fields of study, the interval between matriculation examination and commencement of study, enrolment restrictions, and study patterns. None of these can be expected to remain constant over

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[1] The gymnas is a secondary school with a final examination which one has to pass to be entitled to enter university.

a long period. In the model, therefore, all coefficients may, in principle, vary with time.

II. DESCRIPTION OF THE MODEL.

Each educational process consists of ten activities. The first activity applies to students in their first year of study, the second to those in their second year, and so forth. The study period is considered as a whole, that is, without regarding possible interruptions.

We begin by establishing the number of students to be found in each activity at a certain time. How many of these students will, in the course of a year, graduate from each process, how many will drop out from each process, how many will transfer from one process to another, and how many will be in the various activities the following year?

Admission to restricted fields of study, to non-academic education and to foreign study is included as data, whereas admission to non-restricted fields of study is calculated from the model. Primary admission to non-restricted fields of study refers to the first admission of gymnas graduates. It is made up partly of new graduates, partly of graduates from previous years. An interval of up to three years between gymnas graduation and commencement of study is employed in the model. This is also the case for restricted fields of study, foreign study, and non-academic education.

Secondary admission to non-restricted fields of study (which can be either a positive or a negative figure) covers the students who are redistributed from restricted fields of study or foreign study caused by limited capacity and altered admission policies. The third form of considered admission to non-restricted fields of study is transition from other non-restricted fields of study.

To compute primary admission to non-restricted fields of study, we require information on the size of the graduating gymnas cohorts, $A(t+T)$. (Explanation of the symbols can be found in Appendix 1). We assume that for every gymnas cohort there are certain propensities for certain fields of study, $a_r(t+T)$, i.e., a definite fraction of the cohort wishes to

enter a certain field of study. The quantity, $A_r(t+T)$, is derived as the product of gymnas cohort size and the study propensity:

$$A_r(t+T) = a_r(t+T) \cdot A(t+T)$$

A certain fraction of these begin their studies in the same year as graduation from the gymnas, $k_{0r}(t+T)$, some begin the following year, $k_{1r}(t+T)$, some two years later, $k_{2r}(t+T)$, and the rest begins three years later, $k_{3r}(t+T)$. These fractions are designated here as interval coefficients. The number of graduates from one year's cohort entering a definite field of study in the same year, $Y_r(t+T)$, the year after, $Z_r(t+T)$, two years after, $W_r(t+T)$, and three years after, $U_r(t+T)$, respectively, are calculated as the product of the relevant interval coefficient and the number of students wanting to enter a particular field of study.

$$Y_r(t+T) = k_{0,r}(t+T) \cdot A_r(t+T)$$
$$Z_r(t+T) = k_{1,r}(t+T) \cdot A_r(t+T)$$
$$W_r(t+T) = k_{2,r}(t+T) \cdot A_r(t+T)$$
$$U_r(t+T) = k_{3,r}(t+T) \cdot A_r(t+T)$$

The same method is used to compute the number of gymnas graduates from one year's cohort entering a restricted field of study or a field of study in a foreign country in the same year, the next year, two years later, and three years later. Here the number is computed as the product of the relevant interval coefficient and admission for the pertinent year, $X_r(t+T)$:

$$Y_r(t+T) = l_{0,r}(t+T) \cdot X_r(t+T)$$
$$Z_r(t+T) = l_{1,r}(t+T+1) \cdot X_r(t+T+1)$$
$$W_r(t+T) = l_{2,r}(t+T+2) \cdot X_r(t+T+2)$$
$$U_r(t+T) = l_{3,r}(t+T+3) \cdot X_r(t+T+3)$$

Subsequent to the above calculations, one can compute the number of gymnas graduates from each year who will sooner or later enter the field of study of their first choice, $B(t+T)$:

$$B(t+T) = \sum_{r=1}^{m} (Y_r(t+T)+Z_r(t+T) + W_r(t+T) + U_r(t+T))$$

Assuming that for each gymnas cohort there is a certain total study propensity, $s(t+T)$, we can calculate the number of graduates wanting further academic education, $S(t+T)$:

$$S(t+T) = s(t+T) \cdot A(t+T)$$

If $S(t+T) = B(t+T)$ there is a correspondence between the presumed total study propensity and the proportion of gymnas graduates being admitted to the field of study of their first choice. In the opposite case, we obtain a difference that can be either positive or negative:

$$R(t+T) = S(t+T) - B(t+T)$$

If $R(t+T)$ is positive, we have a number of gymnas graduates not being admitted to the field of study of their first choice. These students must either refrain from further academic education, or places must be made available for them in the restricted fields of study, or they must be distributed among the non-restricted fields of study. This distribution is made with the aid of distribution coefficients, $\gamma_r(t+T)$ and we call this secondary admission.[1)]

The new admissions, $C_r(t+T)$, to the basic courses in non-restricted fields of study are then:

$$C_r(t+T) = Y_r(t+T) + Z_r(t+T-1) + W_r(t+T-2) + U_r(t+T-3) + \gamma_r(t+T)\, R(t+T)$$

of which all but the last quantities represent the primary admission from the last four gymnas cohorts, and the last quantity represents the secondary admissions.

If $R(t+T)$ is negative, the calculated number of entrants to academic educational processes is larger than the number that according to our assumed total study propensity, wants further academic education. On one hand, we may accept this. If so, we accept, as a corrected estimate for the total study propensity, the proportion which is consistent with the calculated number. On the other hand, we may maintain our first estimate by reducing the number of entrants to the basic courses in the non-restricted fields of study. Thereby we assume that all the available places in the restricted fields of study are utilized.

In addition to the new admissions, transition from other fields of study may occur. This is calculated as a fraction, $m_{r,s}(t+T)$, of drop-outs from other fields of study, $M_s(t+T)$.

Admission to an advanced course in a non-restricted field of study is determined as a fraction, $e_{r,s}(t+T)$, of those who have graduated from the basic courses the previous year, $E_s(t+T)$. A possible interval between the basic course and the advanced course is included in the total study period for the advanced course. The formula for calculation of admission to the

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1) For the secondary admission, an interval is not considered.

non-restricted fields of study is:

$$X_r(t+T) = C_r(t+T) + \sum_{s=1}^{p} m_{r,s}(t+T) \cdot M_s(t+T) + \sum_{s=1}^{q} e_{r,s}(t+T) \cdot E_s(t+T),$$

of which the first two quantities refer to the basic courses, and the last refers to the advanced courses.

As mentioned above, admissions to the restricted fields of study and foreign study are included as data. The number of students in the first activity in each educational process is equal to total admissions to the process.

$$N_{r,1}(t+T) = X_r(t+T)$$

The number of students in the other activities is considered as a definite fraction, $c_{r,i-1}(t+T-1)$, of the number of students in the preceding activity of the previous year, $N_{r,i-1}(t+T-1)$:

$$N_{r,i}(t+T) = c_{r,i-1}(t+T-1) \cdot N_{r,i-1}(t+T-1) \qquad (i = 2 \ldots\ldots 10)$$

The number of graduates from each activity is calculated as a definite fraction, $h_{r,i}(t+T-1)$ for graduation in autumn, and $v_{r,i}(t+T-1)$, for graduation in spring, of the total number of students who were enrolled in the activity at the beginning of the year, $N_{r,i}(t+T-1)$.

Thus the total number of graduates from an educational process during an academic year, $E_r(t+T)$, is the sum of the number of graduates from each activity.

$$E_r(t+T) = \sum_{i=1}^{10} (h_{r,i}(t+T-1) + v_{r,i}(t+T-1)) \cdot N_{r,i}(t+T-1)$$

The number of drop-outs from the various activities of an educational process during the academic year is calculated as a definite fraction, $f_{r,i}(t+T-1)$, of those enrolled at the beginning of the year. The total number of drop-outs from an educational process during one year is the sum of drop-outs from each activity.

$$M_r(t+T) = \sum_{i=1}^{10} f_{r,i}(t+T-1) \cdot N_{r,i}(t+T-1)$$

III. ON DATA AND RESULTS.

The following magnitudes can be calculated with the aid of the model:

1. Admission to the courses in non-restricted fields of study for each year of the prediction period, $X_r(t+T)$.
2. The total number of students in each activity for each year of the prediction period, $N_{r,i}(t+T)$.
3. The number of graduated candidates from each educational process for each year of the prediction period $E_r(t+T)$.
4. The total number of drop-outs from each educational process for each year of the prediction period, $M_r(t+T)$.

The model requires the following data:

A. The number of students in each activity in the basis year, $N_{r,i}(t)$.

B. Data concerning the transition from gymnas to the educational processes.

1. The total number of gymnas graduates for each year in the prediction period $A(t+T)$. Here we can either use already prepared predictions, or calculate an estimate of graduates as a subprocess of the model.
2. The number of admissions to restricted fields of study, to foreign study and to non-academic education for every year in the prediction period, $X_r(t+T)$.
3. Total study propensity for each graduating cohort from the gymnasium, $s(t+T)$.
4. Particular propensities for basic courses in the non-restricted fields of study for each graduating gymnas cohort, $a_r(t+T)$.
5. The coefficients for interval between gymnas graduation and commencement of each educational process, $k_{j,r}(t+T)$ and $l_{j,r}(t+T)$.
6. Distribution coefficients for secondary admission to basic courses in non-restricted fields of study.

C. Data concerning study patterns.

1. The coefficients for transition between the non-restricted fields of study for each year in the prediction period, $m_{r,s}(t+T)$.
2. The coefficients for advanced-level education (transition from a basic course to an advanced course in non-restricted fields of study) for each year in the prediction period, $e_{r,s}(t+T)$.

3. Continuation coefficients (transition from one activity to the next in the same educational process) for each year of the prediction period, $c_{r,i}(t+T)$.

4. Graduation coefficients (graduation in respectively autumn and spring from each activity) for each year in the prediction period, $h_{r,i}(t+T)$ and $v_{r,i}(t+T)$.

5. Drop-out coefficients (drop-out from each activity) for each year of the period, $f_{r,i}(t+T)$.

We have made five numerical examples based on different data concerning the transition from gymnas to the educational processes, while the number of students in each activity in the basis year and the data concerning study patterns are the same in all examples.

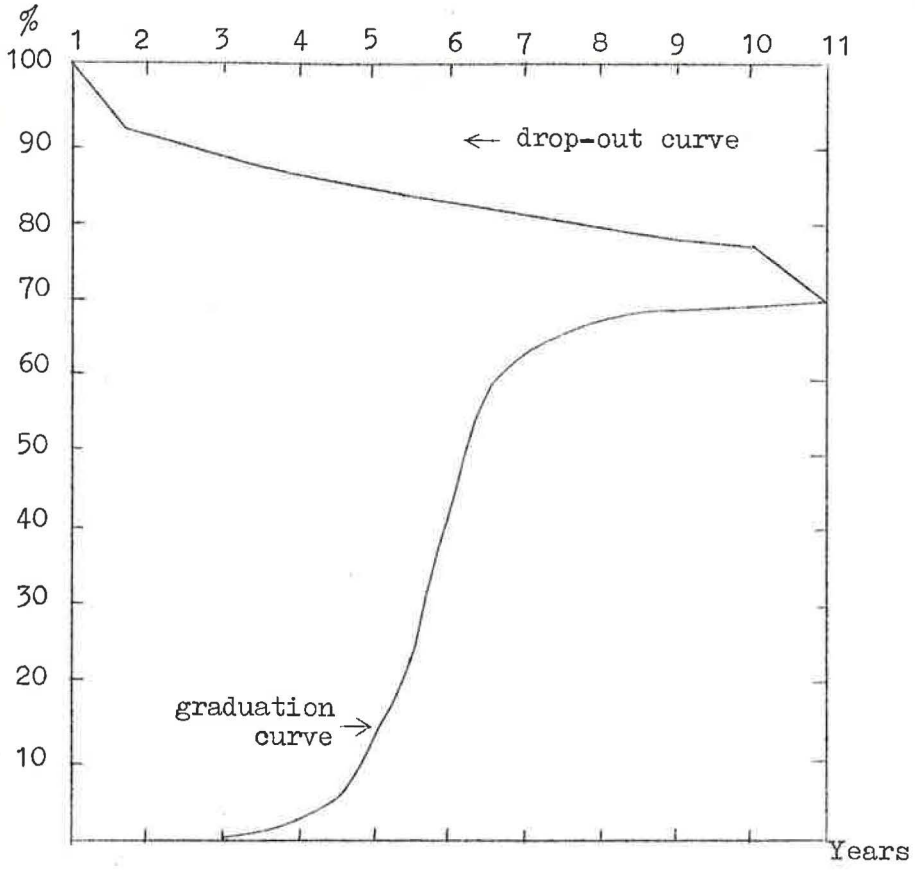
The figures for admission to restricted fields of study, to foreign study and to non-academic education were determined on the basis of all available information about present and future enrolment capacities. The admission figures for these educational processes are the same in all the numerical examples. The estimates concerning restricted fields of study, foreign study, and non-academic education will consequently be the same in all the numerical examples.

The basis year for the calculations is 1964, and the prediction period is 1965 - 1985. The number of students and their distribution among the activities in the basis year were determined chiefly on the basis of statistical material on students from the Central Bureau of Statistics.

In order to establish the estimates for the coefficients for the interval between gymnas graduation and commencement of university study, for continuation, for drop-out, and for graduation from the educational processes, we examined the study patterns of the gymnas cohorts of 1951 and 1958. For most of the processes we also collected supplementary information.

On the basis of this information we drew a study profile for each educational process, i.e., a diagram which shows a drop-out curve and a graduation curve.

Study profile for course of study no. X.



On the basis of these study profiles, we determined the coefficients for graduation, continuation, and drop-out.
Since we have relatively little information about transition between the processes, these coefficients are all determined to be zero in these examples.
Special information was gathered in order to establish the coefficients for advanced-level education.

We use two alternative methods for calculating admissions to the basic courses in non-restricted fields of study. In alternative I, the calculations are made from the assumed particular study propensities. In alternative II, we also have secondary admissions because of "bottlenecks" in the system. Lacking information about the redistribution, we calculate with secondary admissions only to social and natural sciences. The distribution coefficients were decided to be proportional to the primary admission to these studies.

The first numerical example.

The size of the gymnas cohorts 1966-1985 was estimated from the prediction prepared by The Norwegian Research Council for Science and the Humanities, alternative II (see appendix 3) plus a supplement of 10 percent, representing graduates from the commercial gymnas.
The total study propensity was assumed to be 0,50 each year of the period.

The particular study propensities for the basic courses in non-restricted fields of study were estimated on the basis of data from 1965 and were assumed to be the same for each gymnas cohort in the prediction period:

	%
Humanities	16.40
Law	4.00
Theology	0.90
Natural sciences	9.00
Economics	2.00
Psychology	2.00
Pedagogy	0.80
Sociology	0.30
Political science	0.60
T o t a l	36.00

Some main results are shown in Table 1 and Table 2.

Table 1. Number of students. First numerical example.

		1970		1975		1980		1985	
	1964	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of students in:									
Humanities	5293	9472	9278	12152	11984	14575	14527	16827	16981
Law	1190	2704	2704	3499	3499	4202	4202	4832	4832
Theology	504	701	701	859	859	1026	1026	1175	1175
Natural sciences	3791	7002	5812	8919	7812	10712	10435	12364	13257
Economics	437	1338	1073	1712	1518	2053	2031	2353	2559
Psychology	516	1037	851	1258	1126	1494	1480	1716	1872
Pedagogy	182	291	230	360	331	434	437	493	553
Sociology	54	168	136	215	192	256	254	294	320
Political science	159	360	295	484	427	578	566	666	718
Number of students in non-restricted fields of study	12126	23082	21089	29458	27748	35330	34958	40720	42226
Number of students in:									
Medicine	972	1346		2125		2219		2219	
Dentistry	197	586		829		910		910	
Pharmacy	142	186		198		357		372	
Veterinary medicine	156	210		224		224		224	
Agriculture	343	408		420		420		420	
Business administr. and economics	421	674		674		674		674	
Engineering	2178	3631		3975		4488		4645	
Architecture	282	416		494		496		496	
Number of students in restricted fields of study	4691	7457		8939		9788		9960	
Total number of students in academic fields of study in Norway	16817	30539	28546	38397	36687	45118	44746	50680	52227
Number og students in academic fields of study in foreign countries	3659	3510		3392		3470		3494	
Number of students in non-academic education	6068	10428		10453		10536		10534	

Table 2. Average number of graduates per year from academic fields of study. First numerical example.

	1964/65-1969/70		1970/71-1974/75		1975/76-1979/80		1980/81-1984/85	
	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt. II
Number of graduates from advanced courses in:								
Humanities	125	125	273	272	389	383	478	473
Law	122	122	287	287	382	382	463	463
Theology	41	41	61	61	73	73	89	89
Natural sciences	227	226	482	415	638	532	779	734
Economics	38	38	134	108	176	150	214	208
Psychology	41	41	94	79	112	95	136	131
Pedagogy	8	8	15	12	20	17	24	24
Sociology	5	5	16	13	21	18	26	25
Political science	6	6	29	24	43	35	52	50
Medicine in Norway	141		170		306		325	
Medicine in foreign countries	116		147		145		159	
Dentistry in Norway	46		105		149		164	
Dentistry in foreign countries	79		58		65		65	
Pharmacy	26		33		35		63	
Veterinary medicine	25		33		36		36	
Agriculture	122		149		151		151	
Business administration and economics in Norway	145		190		190		190	
Business administration and economics in foreign countries	83		95		104		104	
Engineering in Norway	407		645		708		799	
Engineering in foreign countries	251		204		171		169	
Architecture in Norway	54		75		89		89	
Architecture in foreign countries	19		21		14		13	

The second numerical example.

We used the same estimates for the size of the gymnas cohorts as in the first numerical example.

The total study propensity was assumed to be the same as in the first numerical example.

The particular propensities for the social sciences were assumed to increase from 1966-1985, while the particular propensity for humanities was assumed to decrease correspondingly.

The particular propensity for:

Economics	increases	from	2.00	in	1966	to	3.00	in 1975
Psychology	"	"	2.00	"	"	"	3.00	" "
Sociology	"	"	0.30	"	"	"	2.00	" "
Political science		"	0.60	"	"	"	2.00	" "
Pedagogy		"	0.80	"	"	"	2.00	" "
Humanities	decreases	"	16.40	"	"	"	10.10	" "

From 1975 to 1985 the propensities were assumed to be as in 1975.

The other particular propensities were assumed to be as in the first numerical example.

Some main results are shown in Table 3 and Table 4.

Table 3. Number of students. Second numerical example.

	1964	1970		1975		1980		1985	
		Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of students in:									
Humanities	5293	8934	8786	9788	9651	10101	10048	11240	11326
Law	1190	2704	2704	3499	3499	4202	4202	4832	4832
Theology	504	710	710	859	859	1026	1026	1175	1175
Natural sciences	3791	6958	5830	8882	7906	10743	10509	12450	13125
Economics	437	1424	1146	2180	1970	3000	2976	3527	3761
Psychology	516	1186	980	1718	1566	2337	2320	2756	2941
Pedagogy	182	332	251	653	606	1037	1044	1216	1327
Sociology	54	356	288	959	884	1494	1486	1768	1891
Political Science	159	510	418	1082	985	1648	1627	1986	2109
Number og students in non-restricted fields of study	12126	23114	21113	29620	27926	35588	35238	40950	42487
Number of students in restricted fields of study.[1]	4691	7457		8939		9788		9960	
Total number of students in academic fields of study in Norway	16817	30561	28570	38559	36865	45376	45026	50910	52447
Number of students in academic fields of study in foreign countries[1]	3659	3510		3392		3470		3494	
Number of students in non-academic education[1]	6068	10428		10453		10536		10534	

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education is the same as in the first numerical example.

Table 4. Average number of graduates per year from academic fields of study. Second numerical example.

	1964/65-1969/70		1970/71-1974/75		1975/76-1979/60		1980/81-1984/85	
	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of graduates from advanced courses in:								
Humanities	125	125	272	271	354	350	359	356
Law	122	122	288	288	382	382	463	463
Theology	41	41	61	61	73	73	89	89
Natural sciences	227	226	480	415	634	537	778	741
Economics	38	38	138	111	215	187	308	302
Psychology	42	42	105	89	149	128	215	209
Pedagogy	8	8	16	13	32	28	55	55
Sociology	5	5	25	21	77	67	138	135
Political science	6	6	30	25	76	64	134	129

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.

The third numerical example.

The size of the gymnas cohorts 1966-1985 was estimated from the prediction prepared by the Norwegian Research Council for Science and the Humanities, alternative I (see appendix 3), plus a supplement of 10 percent representing graduates from the commercial gymnas.

The total study propensity was assumed to increase from 0,50 in 1966 to 0,60 in 1975 and to be 0,60 from 1975-1985. The particular study propensities were assumed to be the same as in the first numerical example.

Some main results are shown in Table 5 and Table 6.

Table 5. Number of students. Third numerical example.

	1964	1970		1975		1980		1985	
		Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of students in:									
Humanities	5293	9534	9485	12755	13240	16058	17268	19245	21102
Law	1190	2719	2719	3668	3668	4633	4633	5530	5530
Theology	504	715	715	907	907	1139	1139	1352	1352
Natural sciences	3791	7039	6612	9334	12057	11786	19239	14126	25329
Economics	437	1345	1252	1800	2440	2271	3823	2698	4901
Psychology	516	1046	1000	1332	1818	1661	2771	1976	3571
Pedagogy	182	292	287	379	570	480	840	568	1068
Sociology	54	169	160	228	308	285	474	338	610
Political Science	159	363	344	509	668	639	1047	763	1369
Number of students in non-restricted fields of study	12126	23222	22574	30912	35676	38952	51234	46596	64832
Number of students in restricted fields of study[1]	4691	7457		8939		9788		9960	
Total number of student in academic fields of study in Norway	16817	30679	30031	39851	44615	48740	61022	56556	74792
Number of students in academic fields of study in foreign countries[1]	3659	3510		3392		3470		3494	
Number of students in non-academic education[1]	6068	10428		10453		10536		10534	

1) The number of students in restricted fields of study, in academic fields of study in foreign countries and in non-academic education are the same as in the first numerical example.

Table 6. Average number of graduates per year from academic fields of study. Third numerical example.

	1964/65-1969/70		1970/71-1974/75		1975/76-1979/80		1980/81-1984/85	
	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of graduates from advances coursed in:								
Humanities	125	125	273	272	396	394	511	526
Law	122	122	288	288	396	396	506	506
Theology	41	41	61	61	76	76	98	98
Natural sciences	227	225	482	431	655	732	843	1310
Economics	38	38	134	117	183	227	235	388
Psychology	41	41	94	84	116	138	149	240
Pedagogy	8	8	15	14	20	28	26	45
Sociology	5	5	16	14	22	27	28	46
Political science	6	6	29	25	44	49	57	88

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.

The fourth numerical example.

The size of the gymnas cohorts 1966-1985 was assumed to be the same as in the third numerical example. The study propensities (both total and particular) were assumed to be the same as in the first numerical example. Some main results are shown in Table 7 and Table 8.

Table 7. Number of students. Fourth numerical example.

	1964	1970		1975		1980		1985	
		Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of students in:									
Humanities	5293	9534	9351	12755	12660	16058	16173	19245	19661
Law	1190	2719	2719	3668	3668	4633	4633	5530	5530
Theology	504	715	715	907	907	1139	1139	1352	1352
Natural Sciences	3791	7039	5904	9334	8651	11786	12505	14126	16602
Economics	437	1345	1092	1800	1701	2271	2453	2698	3219
Psychology	516	1046	871	1332	1272	1661	1795	1976	2360
Pedagogy	182	292	234	379	376	480	532	568	699
Sociology	54	169	139	228	217	285	307	338	403
Political science	159	363	302	509	478	639	682	763	901
Number of students in non-restricted fields of study	12126	23222	21327	30912	29930	38952	40219	46596	59727
Number of students in restricted fields of study[1]	4691	7457		8939		9788		9960	
Total number of students in academic fields of study in Norway	16817	30679	28784	39851	38869	48740	50007	56356	60687
Number of students in academic fields of study in foreign countries.[1]	3659	3510		3392		3470		3494	
Number of students in non-academic education[1]	6068	10428		10453		10536		10534	

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education are the same as in the first numerical example.

Table 8. Average number of graduates per year from academic fields of study. Fourth numerical example.

	1964/65-1969/70		1970/71-1974/75		1975/76-1979/80		1980/81-1984/85	
	Alt.I	Alt.II	Alt.I	Alt.II	Alt. I	Alt.II	Alt.I	Alt.II
Number of graduates from advanced courses in:								
Humanities	125	125	273	272	395	389	512	509
Law	122	122	288	288	396	396	506	506
Theology	41	41	61	61	76	76	98	98
Natural Sciences	227	226	482	417	655	566	843	862
Economics	38	38	134	108	183	165	235	248
Psychology	41	37	94	80	116	103	149	156
Pedagogy	8	8	15	12	20	19	26	28
Sociology	5	5	16	13	22	20	28	30
Political science	6	6	29	24	44	38	57	58

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.

The fifth numerical example.

The size of the gymnas cohorts 1966-1973 was assumed to be the same as in the first numerical example. The size of the gymnas cohorts 1974-1985 was assumed to be 25 percent of the $19\frac{1}{2}$ years old persons plus a supplement of 10 percent representing graduates from the commercial gymnas. The study propensities (both total and particular) were assumed to be the same as in the first numerical example. Some main results are shown in Table 9 and Table 10.

Table 9. Number of students. Fifth numerical example.

	1964	1970		1975		1980		1985	
		Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of students in:									
Humanities	5293	9472	9279	11991	11795	13033	12797	13111	12851
Law	1190	2704	2704	3464	3464	3767	3767	3755	3755
Theology	504	710	710	846	846	903	903	894	894
Natural sciences	3791	7002	5812	8822	7574	9645	8279	9679	8069
Economics	437	1338	1073	1693	1468	1825	1562	1809	1501
Psychology	516	1037	851	1232	1073	1304	1112	1298	1081
Pedagogy	182	291	230	358	317	385	323	379	316
Sociology	54	168	136	212	184	224	191	222	185
Political Science	159	360	295	477	411	512	440	512	427
Number of students in non-restricted fields of study	12126	23082	21090	29095	27132	31598	29374	31659	29079
Number of students in restricted fields of study[1]	4691	7457		8939		9788		9960	
Total number of students in academic fields of study in Norway	16817	30539	28547	38034	36071	41386	39162	41619	39039
Number of students in academic fields of study in foreign countries[1]	3659	3510		3392		3470		3494	
Number of students in non-academic education[1]	6068	10428		10453		10536		10534	

1) The number of students in restricted fields of study, in academic fields of study in foreign countries, and in non-academic education are the same as in the first numerical example.

Table 10. Average number of graduates per year from academic fields of study. Fifth numerical example.

	1964/65-1969/70		1970/71-1974/75		1975/76-1979/80		1980/81-1984/85	
	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II	Alt.I	Alt.II
Number of graduate from advanced courses in:								
Humanities	125	125	273	272	389	383	461	455
Law	122	122	287	287	381	381	427	427
Theology	41	41	61	61	73	73	82	82
Natural Sciences	227	226	482	416	638	530	735	643
Economics	38	38	134	108	176	149	197	171
Psychology	41	41	94	79	112	94	125	109
Pedagogy	8	8	15	12	20	17	22	19
Sociology	5	5	16	13	21	18	24	21
Political Science	6	6	29	24	43	35	49	43

The number of graduates from restricted fields of study in Norway and from foreign countries are the same as in the first numerical example.

Appendix 1. A survey of symbols used in the model.

$r = s = 1$ ..... 58 educational processes.

$i = 1$ ..... 10 activities.

$j = 0$ ..... 3 years between gymnas graduation and commencement of study.

t = basis year.

$T = 1$ ..... 21 years in the prediction period.

$A(t+T)$ = The number of gymnas graduates in the year t+T.

$A_r(t+T)$ = The number of gymnas graduates in the year t+T wanting to enter a certain educational process.

$a_r(t+T)$ = Propensity for a certain educational process for the t+T-cohort.

$Y_r(t+T)$ = The number of gymnas graduates from the t+T-cohort entering a certain educational process the same year.

$Z_r(t+T)$ = The number of gymnas graduates from the t+T-cohort entering a certain educational process the following year.

$W_r(t+T)$ = The number of gymnas graduates from the t+T-cohort entering a certain educational process two years after.

$U_r(t+T)$ = The number of gymnas graduates from the t+T-cohort entering a certain educational process three years after.

$k_{j,r}(t+T)$ and $l_{j,r}(t+T)$ = The fraction of the t+T-cohort entering a certain educational process j years after.

$X_r(t+T)$ = Admissions to a certain educational process in year t+T.

$B(t+T)$ = The total number of gymnas graduates from the t+T-cohort entering the field of study of their first choice.

$S(t+T)$ = The number of gymnas graduates in the year t+T wanting further academic education.

$s(t+T)$ = Total study propensity for the gymnas graduates in the year t+T.

$R(t+T)$ = The difference between $S(t+T)$ and $B(t+T)$.

$C_r(t+T)$ = New admissions to a certain educational process in the year t+T.

$\gamma_r(t+T)$ = Distribution key for secondary admissions to basic courses in non-restricted fields of study in the year t+T.

$N_{r,i}(t+T)$ = The number of students in activity No. r,i in the year t+T.

$M_r(t+T)$ = The number of drop-outs from a certain educational process during the academic year t+T-1/t+T.

$f_{r,i}(t+T)$ = Transition coefficient for drop-out.

$E_r(t/T)$ = The number of graduates from a certain educational process during the academic year t+T-1/t+T.

$h_{r,i}(t+T)$ and $v_{r,i}(t+T)$ = The fraction of the number of students enrolled in a certain activity at the beginning of the year graduating during the academic year t+T/t+T+1. ($h_{r,i}(t+T)$ for graduation in autumn and $v_{r,i}(t+T)$ for graduation in spring).

$m_{r,s}(t+T)$ = The fraction of the number of drop-outs from an educational process during the academic year t+T-1/t+T entering a certain educational process in the year t+T.

$e_{r,s}(t+T)$ = The fraction of the number of graduates from a basic course of study during the academic year t+T-1/t+T entering an advanced course of study in the year t+T.

$c_{r,i}(t+T)$ = The fraction of the number of students enrolled in activity No. r,i in a certain educational process at the beginning of the year t+T who next year will be enrolled in the activity r,i+1.

$N_r(t+T)$ = The number of students enrolled in a certain educational process in the year t+T.

n = The number of fields of study.

m = The number of academic fields of study.

p = The number of non-restricted fields of study.

q = The number of basic courses in non-restricted fields of study.

Appendix 2. Model of the Post-gymnas Educational System.

1. $A_r(t+T) = a_r(t+T)\ A(t+T)$

2. $Y_r(t+T) = k_{0,r}(t+T)\ A_r(t+T)$
   $Z_r(t+T) = k_{1,r}(t+T)\ A_r(t+T)$
   $W_r(t+T) = k_{2,r}(t+T)\ A_r(t+T)$
   $U_r(t+T) = k_{3,r}(t+T)\ A_r(t+T)$

3. $Y_r(t+T) = l_{0,r}(t+T)\ X_r(t+T)$
   $Z_r(t+T) = l_{1,r}(t+T+1)\ X_r(t+T+1)$
   $W_r(t+T) = l_{2,r}(t+T+2)\ X_r(t+T+2)$
   $U_r(t+T) = l_{3,r}(t+T+3)\ X_r(t+T+3)$

4. $B(t+T) = \sum_{r=1}^{m} (Y_r(t+T) + Z_r(t+T) + W_r(t+T) + U_r(t+T))$

5. $S(t+T) = s(t+T)\ A(t+T)$

6. $R(t+T) = S(t+T) - B(t+T)$

7. $C_r(t+T) = Y_r(t+T) + Z_r(t+T-1) + W_r(t+T-2) + U_r(t+T-3) + \gamma_r(t+T)\ .\ R(t+T)$

8. $M_r(t+T) = \sum_{i=1}^{10} f_{r,i}(t+T-1)\ N_{r,i}(t+T-1)$

9. $E_r(t+T) = \sum_{i=1}^{10} (h_{r,i}(t+T-1) + v_{r,i}(t+T-1))N_{r,i}(t+T-1)$

10. $X_r(t+T) = C_r(t+T) + \sum_{s=1}^{p} m_{r,s}(t+T)\ M_s(t+T) + \sum_{s=1}^{q} e_{r,s}(t+T)\ E_s(t+T)$

11. $N_{r,i}(t+T) = X_r(t+T)$ for $i = 1$

12. $N_{r,i}(t+T) = c_{r,i-1}(t+T-1)\ N_{r,i-1}(t+T-1)$ for $i = 2 \ldots\ldots 10$

13. $N_r(t+T) = \sum_{i=1}^{10} N_{r,i}(t+T)$

14. $N(t+T) = \sum_{r} N_r(t+T)$

Appendix 3.

Estimates on pupil frequencies and gymnas graduate frequencies in the gymnas 1963/64 - 1984/85.[1]

Session	Pupil frequencies in first gymnas cohort[2]		Gymnas graduate frequencies	
	Alternative I	Alternative II	Alternative I	Alternative II
1963/64	19.3			
1964/65	21.6			
1965/66	22.9		17.4	
1966/67	24.3	23.9	19.4	
1967/68	25.7	24.9	20.6	
1968/69	27.1	25.9	21.9	21.5
1969/70	28.5	26.9	23.1	22.4
1970/71	29.9	27.9	24.4	23.3
1971/72	31.3	28.9	25.7	24.2
1972/73	32.7	29.9	26.9	25.1
1973/74	34.1	30.9	28.2	26.0
1974/75	35.5	31.9	29.4	26.9
1975/76	36,9	32,9	30.7	27.8
1976/77	38.3	33.9	32.0	28.7
1977/78	39.7	34.9	33.2	29.6
1978/79	41.1	35.9	34.5	30.5
1979/80	42.5	36.9	35.7	31.4
1980/81	43.9	37.9	37.0	32.3
1981/82	45.3	38.9	38.3	33.2
1982/83	46.7	39.9	39.5	34.1
1983/84	48.1	40.9	40.8	35.0
1984/85	49.5	41.9	42.0	35.9

1) The number of gymnas graduates in the last three years has been about 90 % of the number of pupils in the first gymnas cohort two years earlier. The increase in the pupil frequencies the last eight years has been 1.4 percent per year, on an average. In alternative I, this increase is assumed to continue. In alternative II, the assumption is an increase of 1.0 percent per year.

2) First class in the 3-year gymnas, or third class in the 5-year gymnas.

Prediction on the number of gymnas graduates, according to the above assumptions.

Year	Alt. I	Alt. II	Year	Alt. I	Alt. II
1966	11.536	11.536	1976	18.911	17.125
1967	12.397	12.397	1977	19.584	17.564
1968	12.690	12.690	1978	20.302	18.100
1969	13.206	12.965	1979	20.924	18.498
1970	13.745	13.328	1980	21.349	18.777
1971	14.525	13.870	1981	21.998	19.204
1972	15.595	14.685	1982	22.927	19.874
1973	16.350	15.256	1983	23.748	20.501
1974	17.165	15.826	1984	24.616	21.117
1975	18.155	16.611	1985	25.787	22.042