Journal of Informetrics 13 (2019) 679–694

Contents lists available at ScienceDirect

Journal of Informetrics

journal homepage: www.elsevier.com/locate/joi

Regular article

Measuring scientific contributions with modified fractional counting

Gunnar Sivertsen[a], Ronald Rousseau[b,c], Lin Zhang[c,d,*]

[a] *Nordic Institute for Studies in Innovation, Research and Education, Oslo, Norway*
[b] *University of Antwerp, Faculty of Social Sciences, B-2020 Antwerpen, Belgium*
[c] *Centre for R&D Monitoring (ECOOM) and Dept. MSI, KU Leuven, Belgium*
[d] *School of Information Management, Wuhan University, China*

ARTICLE INFO

Article history:
Received 28 August 2018
Received in revised form 20 March 2019
Accepted 20 March 2019

Keywords:
Scientific contributions
Research output
Bibliometrics
Full counting
Fractional counting
Harmonic counting
Modified fractional counting
Sensitivity parameter
Performance indicators
Rankings
Leiden Ranking
Web of Science
Norwegian Science Index

ABSTRACT

We develop and propose a new counting method at the aggregate level for contributions to scientific publications called modified fractional counting (MFC). We show that, compared to traditional complete-normalized fractional counting, it eliminates the extreme differences in contributions over time that otherwise occur between scientists that mainly publish alone or in small groups and those that publish with large groups of co-authors. As an extra benefit we find that scientists in different areas of research turn out to have comparable average contributions to scientific articles. We test the method on scientists at Norway's largest universities and find that, at an aggregate level, it indeed supports comparability across different co-authorship practices as well as between areas of research. MFC is thereby useful whenever the research output from institutions with different research profiles are compared, as e.g., in the Leiden Ranking. Finally, as MFC is actually a family of indicators, depending on a sensitivity parameter, it can be adapted to the circumstances.

1. Introduction

The statistics, evaluation, and funding of research is often based on a bibliometric quantification of the contributions of different actors (authors, institutions, countries). Yet, counting methods not only represent purely bibliometric or mathematical problems: they can, moreover, strongly affect decision-making and resource allocation in research. Our study focuses on one of the most widespread applications of bibliometrics: methods for counting scientific articles. On an empirical basis, we ask how well the traditional counting methods represent the reality of scientific contributions and we offer a new solution, called modified fractional counting (MFC).

The most well-known and widespread counting methods based on article data, are full counting and fractionalized counting. Full counting gives each contributing author one credit, i.e., five authors equals five credits. Fractional counting assigns a fraction of one credit to each author (Egghe, Rousseau, & Van Hooydonk, 2000; Osório, 2018; Waltman & van Eck,

* Corresponding author at: Centre for R&D Monitoring (ECOOM) and Dept. MSI, KU Leuven, Belgium.
E-mail address: zhanglin_1117@126.com (L. Zhang).

https://doi.org/10.1016/j.joi.2019.03.010
1751-1577/

Table 1
Counting methods for scientific publications.

	Each contributor receives some credit	Some contributors may not receive any credit
Credits are natural numbers	Complete, also known as full or normal count	First author count; Major contribution count
Credits may be fractions, summing to one	Complete-normalized fractional counting; Harmonic counting	Fractionalized major contribution count
Credits may even be irrational numbers (never larger than one), possibly summing to a number larger than one	Modified fractional counting (MFC)	(Not relevant)

2015; Waltman, 2016). At the aggregate level, organizations or countries may be credited according to the number of authors affiliated with them, or only once for each unique contributing organization or country.

Both full and fractional counting methods can be useful as they provide information from different perspectives, e.g., participation (full counting) versus contribution (fractional counting) (Moed, 2005). For several reasons, including the need for normalized indicators across fields of research, fractional counting is often preferred in professional and scientific bibliometrics operating at an aggregate level (Waltman, 2016). From an aggregate perspective, fractional counting adds up to the same number of articles as are in the data, which provides balance, consistency, and precision in advanced field-normalized bibliometric measurements (Waltman & van Eck, 2015). Full counting is more widespread at the individual level. The h-index introduced by Hirsch (2005) is a good example of full counting. (Although, some h-index variations take the number of authors into account, such as e.g., (Chai, Hua, Rousseau, & Wan, 2008)

Our study aims at improving counting methods at the aggregate level. The focus is thereby on fractional counting and how it represents scientific contributions. We will, however, also present the results from full counting throughout this study since we are investigating the effects of intermediate solutions between full and fractional counting.

Counting methods for scientific articles can also be classified according to whether or not every author receives credit. Combining traditional classification systems and the new MFC solution creates the alternatives shown in Table 1, which is an elaboration of a similar table presented and discussed in Rousseau, Egghe, and Guns (2018).

In this study, we see articles, and how they are counted, as representations of contributions to *scientific work*, not just as contributions to the *scientific literature*. Accordingly, we agree that a counting procedure can be seen as an estimation method to determine contributions of scientists or, on a higher level, institutions or countries (Egghe et al., 2000). Therefore, we only focus on methods that provide some credit to all authors, eliminating methods in the last column of Table 1. Moreover, we do not focus on individual authors – which could imply taking the order in the byline into account – but on a higher aggregation level, such as organizations or countries, where we find that the order of authors do not affect the results. For this reason, our starting point is giving equal credit to co-authors.

Table 1 separates between so-called complete-normalized fractional counting, by which each co-author receives 1/N of one credit, and harmonic counting, a seldom-used variant of fractional counting, where the rank of co-authors is taken into account to weight their contribution. For our purposes, the term "fractional counting" generally refers to complete-normalized fractional counting, although we have included a brief analysis of harmonic counting in our results in Section 8.

Counting methods are important because they are known to measure performance differently and result in different rankings (Aksnes, Schneider, & Gunnarsson, 2012; Egghe et al., 2000; Gauffriau & Larsen, 2005; Gauffriau, Larsen, Maye, Roulin-Perriard, & von Ins, 2007; Martin, 1994). A well-known example is provided by the CWTS Leiden Ranking[1] – a worldwide ranking system for universities based on scientific contribution and impact. The default ranking method is complete-normalized fractional counting. The world's largest universities are ranked according to the volume of scientific contributions. By this counting method, Zhejiang University and Shanghai Jiao Tong University are ranked third and fourth in 2018. However, in switching to the full counting alternative by unticking "Calculate impact indicators using fractional counting", the two universities are now ranked in the opposite order as numbers nine and ten. In our view, it is not sufficient to say that the first measurement represents contribution while the second represents participation. Both are perceived and used as indicators of scientific output at the aggregate level. Our study will demonstrate that the different results may just as well reflect different research profiles as different tendencies to contribute to research, be it at the local level or world-wide.

As stated initially, the starting point and concern of our study is that bibliometric counting methods not only matter to the field of bibliometrics, they also matter in real life because they provide feedback and incentives to scientists. Full counting stimulates collaboration in research and the possible addition of more (unnecessary) authors while fractional counting provides balanced and precise data, but it can also act as a disincentive to collaboration (Bloch & Schneider, 2016). Complete-normalized fractional counting has been used as the basis for official statistics for a long time, e.g., in the annual "Science & Engineering Indicators" report by the National Science Foundation (USA)[2] and "The Science, Research and Innovation Performance of the EU" (SRIP) report by the European Commission.[3] Additionally, several European countries use

[1] http://www.leidenranking.com/
[2] https://www.nsf.gov/statistics/2018/nsb20181/.
[3] https://ec.europa.eu/info/sites/info/files/srip-report-full_2018_en.pdf.

bibliometric indicators in their university funding formulas (Jonkers & Zacharewicz, 2015). For example, Flanders (Belgium) provides funding at the institutional level based on the full counting of articles (Debackere & Glänzel, 2004), as does Croatia and Estonia (Debackere, Arnold, Sivertsen, Spaapen, & Sturn, 2018). Denmark, Finland, Norway, and Sweden use complete-normalized fractional counting to determine institutional funding levels (Sivertsen, 2016a; Vetenskapsrådet, 2014).

Returning to the example of the two Chinese universities going up and down in the Leiden Ranking, it is no surprise that bibliometric counting methods have been questioned on one of the most influential research policy blogs in China.[4] China's size as a science producer versus the US depends on the counting method used. Interestingly, both types of counting methods are questioned in the blog: fractional counting for underestimating scientific contribution, full counting for overestimating it. Consequently, the blog's author, Yishan Wu, launched a challenge: "Can someone boldly propose a new intermediate counting method between full counting and fractional counting?"

In this study, we do not follow the habit of stating that fractional counting is probably preferable for most purposes. Instead, we admit that the two alternatives may seem confusing from perspectives outside bibliometrics, such as the policy-making perspective or the perspective of individual scientists who see their publications counted as fractions. We acknowledge that bibliometrics is not merely used to represent and model scientific literature. The use of bibliometrics is widespread as a way to represent, support, and assess real-life research activity that has been, or will be, reported in the scientific literature. Just as references are used to study citation impact or the influence of research, publications are used to study outputs of research, research profiles, collaboration, and so on. Bibliometrics needs to take the reality of scientific work into account. Hence, we come to the question:

How does the full and fractional counting of articles represent real-world contributions to scientific results at an aggregate level?

Another reason for asking this question is that the empirical evidence revealed in our study shows that:

- With full counting, scientific fields and departments whose scientists frequently publish with a high number of co-authors seem to contribute more.
- With fractional counting, fields and departments whose scientists mostly publish alone or with a small number of co-authors seem to contribute more.

The same observations have been made at an aggregate level (Aagaard, Bloch, & Schneider, 2015; Piro, Aksnes, & Rørstad, 2013). For us, this leads to another question that has not yet been asked in the bibliometric literature:

Can fractional counting methods be modified so that their results lead to comparable average contributions across all fields and all co-authorship practices?

This is the core question of our study. We will argue that Modified Fractional Counting (MFC) provides an affirmative answer to this question.

2. Representing contributions to scientific work in publications

As stated in the introduction, we see articles, and how they are counted, as representations of contributions to *scientific work*, not just as contributions to the *scientific literature*. In this section, we present more reasons for modifying a well-established fractional counting procedure. We think a counting method should not only try to represent the data in the best possible way, but also be valid with regard to what is measured: contributions to scientific work.

Doing research is not the same as writing an article. A publication is simply a "formalized" representation of contributions to scientific results. With increasing collaboration in research, and increasing numbers of authors per article, studying how to replace the traditional one-author model with a contributor model to show how a scientific work is created, has become a pressing issue (Cronin, 2001; Rousseau et al., 2018, p. 32). We need to be open-minded towards finding out how collaborative work in science is actually practiced and then represented in the bibliographical information of an article. Although there may be gains resulting from research collaboration, these gains also come with costs (Katz & Martin, 1997). Representing scientific contributions as fractions of articles depending on the number of authors may not be a good solution. The contributions of the collaborating researchers may be overlapping and not the result of disjoint actions. Collaboration in itself demands coordinating the contributions. In biomedical journals, such as *BMJ*,[5] authors are asked to declare their roles and responsibilities in the research that took place before publishing, not just their contributions to the writing. And even when it comes to writing and publishing the final version, overlapping responsibilities may be the rule. For instance, the "Recommendations for the Conduct, Reporting, Editing, and Publication of Scholarly Work in Medical Journals" by the International Committee of Medical Journal Editors (ICMJE), which BMJ contributes to and follows,[6] require that each author must not only make a substantial contribution to the work but also approve the final version for publication and agree to be "accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved". In addition, but not mentioned as a requirement from ICMJE, come the usual

[4] http://blog.sciencenet.cn/blog-1557-1122817.html.
[5] https://www.bmj.com/about-bmj/resources-authors/article-submission/authorship-contributorship.
[6] http://www.icmje.org/news-and-editorials/updated_recommendations_dec2017.html.

tasks of organizing the project formally, creating the infrastructure, providing funding, and reporting back to the funding organizations.

The recommendations of the ICMJE seem incompatible with fractional counting as a bibliometric practice. It is, however, common knowledge and well-documented (Marušić, Bošnjak, & Jerončić, 2011) that the ICMJE recommendations are seldom followed completely and often even disregarded. But this is not an argument in favor of fractional counting. Rather, these recommendations remind us that a collaborative research process would not be possible without interaction, influence, and agreements among the contributors. These agreements have consequences for receiving co-authorship credit.

The contributions of scientists to co-authored publications may not be realistically represented by the fraction given by dividing the publication by the number of co-authors. This is the motivation behind our design and testing of the MFC method. Still, our concern is not how individual researchers should be credited for publications. We focus on consequences on the aggregate level of using different counting methods for comparing organizations or countries with different research profiles.

3. Data

Our study combines the use of two national level databases. One of them is the 2018 National Citation Report (NCR) for Norway, delivered by Clarivate Analytics, which covers all publications in the Web of Science (WoS) 1981–2017 with at least one author's address in Norway. This database records the total number of authors in each publication, as well as an author's position in the sequence of authors. As a starting point, we selected scientific publications (articles and reviews) from 2011–2017. The second database is the Norwegian Science Index (NSI), a subset of the Current Research Information System in Norway (Cristin), with complete coverage of all peer-reviewed scientific and scholarly publication outputs since 2010, including books, edited volumes, and conference series (Sivertsen, 2018).

All articles and authors in NCR were matched to the corresponding records in NSI. We could then unambiguously relate the bibliographic information in each WoS article to real people, departments, and institutions in Norway. We could also compare the coverage in WoS with each researcher's full set of publications. Since we know from a comparison of the two databases (Sivertsen & Larsen, 2012) that the extent of coverage in WoS differs by field of research, we chose to run the calculations for both databases to see how these differences may affect the measurement of scientific contributions. Although we study counting methods for the aggregate level, we wanted to ensure appropriate conditions for comparing individual level scientific contributions. Hence, we selected only publications by authors who are, or were, employed at Norway's four largest research universities: the University of Bergen, the University of Oslo, the Norwegian University of Science and Technology (Trondheim), and UiT – The Arctic University of Norway (Tromsø). Researchers in all fields at all four universities are given the same resources in time (on average 50 percent) for performing research. The same solution was chosen for the same reasons in an earlier study (Aksnes et al., 2012).

Further, we limited the dataset to publications by scientists with at least two publications in each of the databases in each half (one-year overlapping) of the period 2011–2017 to ensure that the researchers had comparable possibilities to contribute to scientific results during the whole period. This criterion yielded 4048 unique researchers in the WoS-based dataset and 5551 researchers in the NSI-based dataset. We tested a stricter criterion (at least one publication each year, yielding 1410 and 2186 researchers) and a more inclusive criterion (at least one publication in each half of the period, yielding 5553 and 7211 researchers). The results of our calculations were similar for all three selections. We selected the intermediate alternative as most robust.

Our WoS-based dataset has 44,405 unique scientific articles published by 4048 unique persons. To classify the persons by field of research, we used the NSI-classification of publications (84 different fields, e.g., history; political science; neurology; geosciences, which are aggregated into four major areas of research: humanities, social sciences, health sciences, natural sciences and engineering) and allocated each scientist to the field in which they most frequently publish. Only 1.3 percent of the researchers are from the humanities and 4.2 percent from the social sciences in our WoS-based dataset, while 55.4 percent are from the health sciences and 39.1 percent from the natural sciences and engineering. A little less than one third (32.4 percent) are female researchers. The researchers were on average 52 years old in the last year of our study. Almost two thirds of the researchers are professors (66.2 percent), 12.1 percent are assistant professors and 7.8 percent are postdocs. The remaining authors are PhD students, scientists with a PhD but without a formal postdoc appointment, or colleagues from research management offices and technical services.

The NSI-based dataset is an extension of the WoS-based dataset. It includes all WoS-publications but is extended to peer-reviewed research publications not covered by WoS. It has in total 75,271 unique publications in journals, conference proceedings and books that have been published by 5551 unique persons. Among these persons, 9.5 percent are from the humanities, 10.6 percent from the social sciences, 45.0 percent are from the health sciences and 34.9 percent from the natural sciences and engineering. As to the other variables, gender (33.7 percent females), age (52 years old on average) and position (64.8 percent professors), the NSI-based dataset very much resembles the WoS dataset.

4. Grouping scientists according to co-authorship practices

There are persistent and well-known problems with using bibliometric data to compare scientific contributions across fields of research, types of institutions, and individual scientists. Acknowledging these problems, we nevertheless make an

Table 2
The 4048 scientists in our WoS-based dataset divided into 12 groups based on the median number of authors in their publications.

Group name	Number of researchers	Median number of authors in publications	Average number of authors in publications
1	53	1	1.6
2	169	1.5-2	2.6
3	421	2.5-3	3.7
4	550	3.5-4	4.5
5	599	4.5-5	6.2
6	664	5.5-6	7.9
7	459	6.5-7	8.4
8	373	7.5-8	9.4
9	231	8.5-9	10.8
10	153	9.5-10	12.1
15	238	10.5-15	15.5
1000	138	15.5-3017	712.8

Table 3
The 5551 scientists in our NSI-based dataset divided into 12 groups based on the median number of authors in their publications.

Group name	Number of researchers	Median number of authors in publications	Average number of authors in publications
1	594	1	1.3
2	502	1.5-2	2.4
3	640	2.5-3	3.7
4	699	3.5-4	4.8
5	674	4.5-5	6.5
6	720	5.5-6	8.3
7	487	6.5-7	9.0
8	407	7.5-8	10.4
9	249	8.5-9	12.0
10	171	9.5-10	13.4
15	263	10.5-15	17.2
1000	145	15.5-3017	577.9

important working hypothesis in our analysis: We assume that at an aggregate level, scientists in different fields contribute on average to the same extent to scientific results. No field of research is in itself more “important” than another.

The reason for assuming this as a first step, is that all bibliometric counting methods (see Table 1) basically refer only to the bibliographic information about authors in a publication, and not the type of publication or field of research. As an example, fractional counting is based on no other information than the number of authors. Still, this method is widely used to compare organizations or countries with different research profiles. In the introduction we referred to the Leiden Ranking as an example. It is, in practice, based on the same assumption: At an aggregate level, scientists in different fields on average contribute to the same extent to scientific results.

The working hypothesis, which we will return to below, allows us, as a first step, to divide scientists in groups, not by field of research, but by their typical co-authorship practices. According to our assumption, it is the number of authors and not the field of research that determines the outcome of a counting method. Because of this assumption, the used counting method should give a balanced result for different co-authorship practices.

Here, we decided to group researchers according to co-authorship practices and look at their field of research afterwards. However, grouping researchers according to co-authorship practices is not straightforward. Take one scientist’s co-authorship practice across 15 publications as an example: sole author – 1 publication; two authors – 7; three authors – 1; four authors – 2; five authors – 1; ten authors – 1; 16 authors – 1; with the last publication having 40 authors. Due to the extreme differences between these values, we chose to represent the typical co-authorship practice of each scientist by its median (2 in this case), not by its arithmetic mean (6.5 in this case).

We next divided scientists into 12 groups according to the median number of authors in their body of publications. The first 10 groups are named after their median, e.g., the scientists in Group “2″ have publications with a median of 1.5–2 authors. The last two groups are “15″ for medians of 10.5–15 and “1000″ for medians of 15.5-3,017. This last group includes scientists identified as regular contributors to “hyper or mega-authorship” (Cronin, 2001; Kretschmer & Rousseau, 2001). The number of scientists and the average number of authors in their publications for each group are shown in Table 2 (WoS-based data) and Table 3 (NSI-based data).

Co-authorship practices are known to vary by field. With our method, we can demonstrate that they also vary within fields. This is shown for four different fields in Fig. 1 (WoS-based dataset) and four other different fields in Fig. 2 (NSI-based dataset). Contrary to a widespread belief, it is quite clear from our results that fields of research do not carry a unique typical co-authorship practice.

Our results indicate that it is important that counting methods balance between different co-authorship practices. It will not be sufficient to field-normalize or only compare researchers within one field of research at a time. In the next

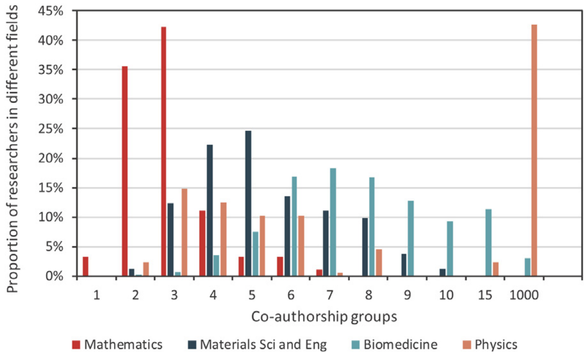

Fig. 1. The distribution of researchers among co-authorship groups in four research fields, using the WoS-based dataset.

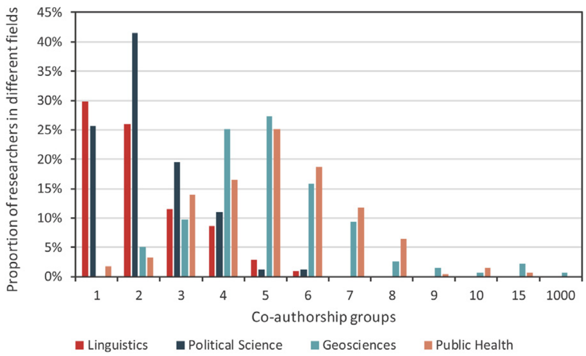

Fig. 2. The distribution of researchers among co-authorship groups in four research fields, using the NSI-based dataset.

sections, we present the methods and results of calculating the outcomes of using different counting methods on the twelve co-authorship groups of researchers.

5. Methods

In principle, the term Modified Fractional Counting (MFC) refers to any method that applies a function to equal fractions. Our approach is based on the general observation that full counting tends to overestimate the contributions to multi-authored publications while fractional counting tends to underestimate the contributions to such publications – see the introduction. Just as Yishan Wu in his blog, we look for an intermediate method between full counting and fractional counting.

As an intermediate method, the square root in fractional counting already had precedence in an empirical study that paved the way for a change to the publications-based performance indicator used to fund research organizations in Norway (Sivertsen, 2016b). The square root of a fraction is interesting because it never exceeds 1, but it adds value to the contribution of each author with an effect that it diminishes as the number of authors increases. These dynamics correspond well to our ideas about added and overlapping contributions in research collaboration (see Section 2 above), and also to the policy need for a counting method that does not provide an incentive to add more (unnecessary) authors.

Nevertheless, we chose to see the square root of the fraction as a particular case of more generally applying a sensitivity parameter based on exponentiation that results in a continuum from complete-normalized fractional counting to full counting. We wanted to perform calculations by using the whole range of possibilities.

We refer to the number k as the sensitivity parameter in MFC methods. Applying MFC using a k-th root is equivalent to giving each author of a publication with N authors a credit equal to $1/\sqrt[k]{N}$. When $k=1$, it represents traditional complete-normalized fractional counting. When $k=2$, the square root is used, and with $k=3$, the cubic root is used. Higher values of k come closer to full counting. The notation MFC_1 represents traditional complete-normalized fractional counting, MFC_2 applies a square root and MFC_3 a cubic root. In addition, we also show results for MFC_4 and MFC_8. The mathematical foundation and implications of this method is presented in the Appendix.

6. Results obtained from calculating and comparing different counting methods

There is no objective best choice for the sensitivity parameter. We decided to first compare $k=2$ (the square root) with the two traditional methods, as k = 2 seems to be a reasonable compromise, and then afterwards show results for higher k values.

Table 4
Average contributions to published research of each co-authorship group measured by three different counting methods using the WoS-based dataset with 4048 researchers.

Group	# Researchers	Average author total	MFC_1	MFC_2	Full counts
1	53	1.6	5.45	5.85	6.51
2	169	2.6	5.24	7.20	10.46
3	421	3.7	5.31	8.68	14.95
4	550	4.5	4.82	8.88	17.22
5	599	6.2	4.51	9.26	20.09
6	664	7.9	4.24	9.46	22.34
7	459	8.4	3.93	9.34	23.66
8	373	9.4	3.25	8.19	22.01
9	231	10.8	3.28	8.68	24.63
10	153	12.1	3.10	8.48	25.27
15	238	15.5	3.00	8.73	28.34
1000	138	712.8	1.99	8.49	139.49

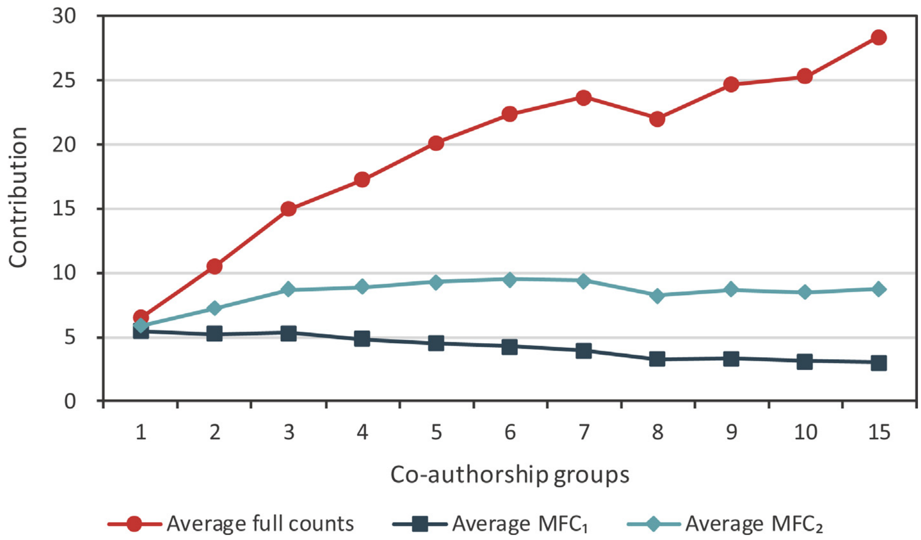

Fig. 3. Distributions of results for co-authorship groups 1–15 based on the last three columns of Table 4.

As explained in Section 4, the scientists in our two datasets are divided into 12 groups based on the median number of authors contributing to their publications. These groups are referred to as “co-authorship groups”. The results derived from the three different counting methods[7] applied to the 4048 scientists in the WoS-based dataset are shown in Table 4.

The distributions for groups 1–15 based on the last three columns in Table 4 are illustrated in Fig. 3.

Our results generally support the idea that fractional counting is preferable to full counting. Full counting overestimates the contributions of scientists involved in publications with many authors. Traditional complete-normalized fractional counting (MFC_1) has the opposite effect, but not to the same degree. MFC_2 uses the square root of fractions for better balance across differing co-authorship practices. Overall contributions appear to be lower in Groups 1 and 2. However, as can be seen in Table 2, these two groups are smaller, and belong mostly to the social sciences and humanities, of which there are few scientists in the WoS-based dataset. We will now perform the same analysis using the NSI-based dataset, which has a more complete coverage of the social sciences and humanities.

The results derived from the three different counting methods on the 5551 researchers in the NSI-based dataset are shown in Table 5.

The distributions for co-authorship groups 1–15 based on the last three columns in Table 5 are illustrated in Fig. 4.

Extending the data source beyond WoS gives somewhat different results. The gain for mainly publishing multi-authored publications by full counting levels out. MFC_2 balances better for groups 1–3, probably because more publications in the social sciences and humanities are now included. However, apart from this, MFC_2 does not lead to the same balanced counting results as in Fig. 3 which was based on WoS-data only. This observation led us to investigate higher values of the sensitive parameter k (see Section 5).

[7] MFC_1 = traditional (complete-normalized) fractional counting. MFC_2 = MFC based on the square roots of fractions. The contributions of each researcher were obtained before averages were calculated for each group. The same applies to Table 5.

Table 5
Average contributions to published research of each co-authorship group measured by three different counting methods using the NSI-based dataset with 5551 researchers.

Group	# Researchers	Average author total	MFC_1	MFC_2	Full counts
1	594	1.3	11.95	12.54	13.51
2	502	2.4	9.16	11.89	16.43
3	640	3.7	8.22	13.12	22.33
4	699	4.8	6.60	12.09	23.55
5	674	6.5	5.23	10.71	23.51
6	720	8.3	4.80	10.73	25.71
7	487	9.0	4.33	10.38	26.84
8	407	10.4	3.56	9.00	24.57
9	249	12.0	3.62	9.60	27.86
10	171	13.4	3.28	9.10	27.96
15	263	17.2	3.11	9.05	30.00
1000	145	577.9	2.59	12.58	169.12

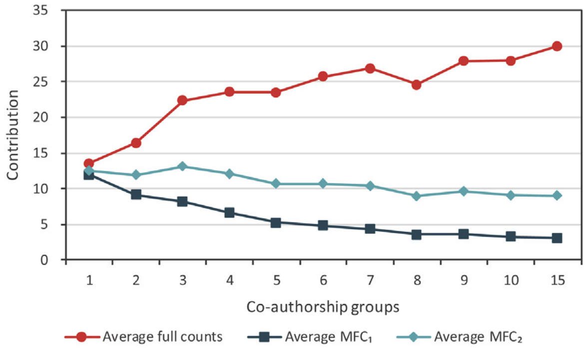

Fig. 4. Distributions of results for co-authorship groups 1–15 based on the last three columns of Table 5.

Table 6
Results for different sensitivity parameter values (k), calculated for the WoS-based data in Table 4.

Group	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full counts
1	5.45	5.85	6.14	6.31	6.50	6.51
2	5.24	7.20	8.62	9.48	10.40	10.46
3	5.31	8.68	11.31	12.98	14.82	14.95
4	4.82	8.88	12.28	14.51	17.01	17.22
5	4.51	9.26	13.54	16.46	19.84	20.09
6	4.24	9.46	14.42	17.91	22.02	22.34
7	3.93	9.34	14.75	18.64	23.31	23.66
8	3.25	8.19	13.32	17.08	21.66	22.01
9	3.28	8.68	14.49	18.84	24.22	24.63
10	3.10	8.48	14.47	19.06	24.82	25.27
15	3.00	8.73	15.49	20.87	27.80	28.34
1000	1.99	8.49	28.68	60.93	132.21	139.49

Table 6 shows the results when applying different values of k in the WoS-based dataset. Again, MFC_1 represents traditional complete-normalized fractional counting, while MFC_2 applies the square root, MFC_3 the cubic root, and so on. As expected, see the mathematical foundations of MFC in the appendix, higher values of k come closer to full counting.

MFC_2 seems to give the most balanced representation of average contributions across different co-authorship practices. Mega- or hyper-authorship (our group denoted as 1000) has been regarded as an exception to the rule that bibliographic information is a reasonable approximation of actual contributions to scientific work (Cronin, 2001). It is, therefore, somewhat surprising that MFC_2 balances even with this group.

Higher values of the sensitivity parameter k do not give more balanced variants of MFC in our WoS-based data. This is more clearly visible in Fig. 5 with the results for co-authorship groups 1–15 from Table 6.

Table 7 shows the same parameters as in Table 6, but now using the NSI-based dataset. With this more complete dataset extending beyond WoS, MFC_3 (using the cubic root of fractions) seems to balance better than the alternatives, although group 1000 is now an exception. The results are more clearly seen in Fig. 6.

So far we may conclude that Modified Fractional Counting balances better between different co-authorship patterns than traditional complete-normalized fractional counting or full counting. More specifically, MFC_2 (using the square root of fractions) seems most adequate when applying WoS-based data while MFC_3 seems adequate as well when applying more

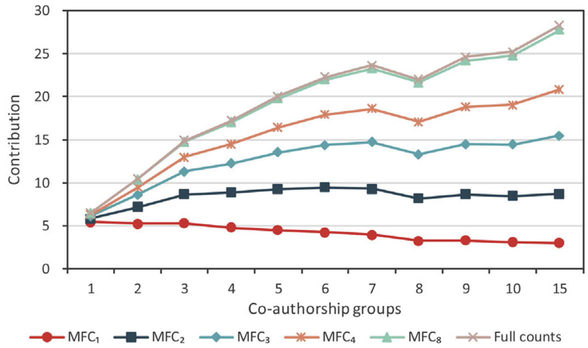

Fig. 5. Results for co-authorship groups 1–15 for the data in Table 6.

Table 7
Results for different sensitivity parameter values (k), calculated for the NSI-based data in Table 5.

Group	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full counts
1	11.95	12.54	12.96	13.22	13.49	13.51
2	9.16	11.89	13.86	15.06	16.34	16.43
3	8.22	13.12	16.98	19.43	22.14	22.33
4	6.60	12.09	16.73	19.80	23.27	23.55
5	5.23	10.71	15.73	19.18	23.20	23.51
6	4.80	10.73	16.45	20.51	25.34	25.71
7	4.33	10.38	16.53	21.01	26.43	26.84
8	3.56	9.00	14.72	18.96	24.17	24.57
9	3.62	9.60	16.16	21.15	27.37	27.86
10	3.28	9.10	15.74	20.90	27.45	27.96
15	3.11	9.05	16.19	21.93	29.41	30.00
1000	2.59	12.58	39.84	79.25	160.99	169.12

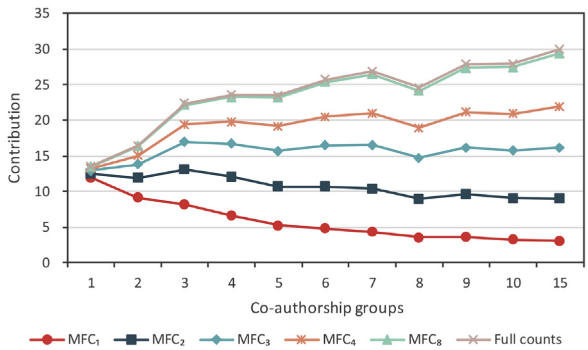

Fig. 6. Results for co-authorship groups 1–15 for the data in Table 7.

complete national or institutional data representing all peer-reviewed research publications. These preliminary conclusions still need to be checked by bringing our analysis down to the level of areas or fields of research.

7. Results from applications at the level of areas and fields of research

We demonstrated in Section 4 that fields of research do not carry a unique typical co-authorship practice. Different co-authorship practices occur within each field, however with different frequencies across fields. Our results from Section 6 indicate that Modified Fractional Counting (MFC) may give a balanced representation of different co-authorship practices. In Section 4, we established a working hypothesis assuming that at an aggregate level, scientists in different fields contribute on average to the same extent to scientific results. Given our results so far, our assumption implies that MFC might also give a balanced representation of scientific contributions across fields of research. We will now investigate to what extent this can be confirmed.

So far, we have used two datasets, one based on WoS and another extended dataset based on NSI. As we now approach the level of areas and fields of research, we must take into consideration that the WoS-based dataset in itself cannot give a balanced representation of the scientific contributions across all fields because it is biased towards the health sciences and the natural sciences and engineering (Sivertsen & Larsen, 2012). This bias is measurable within our data: Focusing on

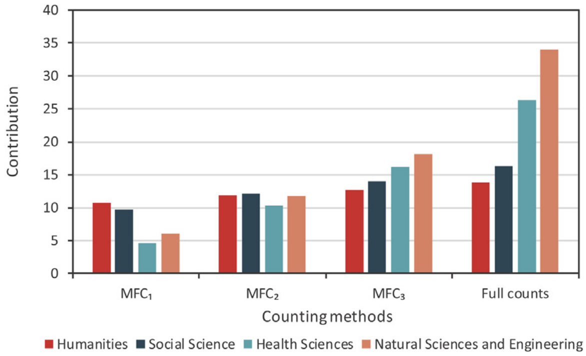

Fig. 7. Average contributions to published research of researchers in four major areas of research using four different counting methods and the NSI-based dataset with 5551 researchers.

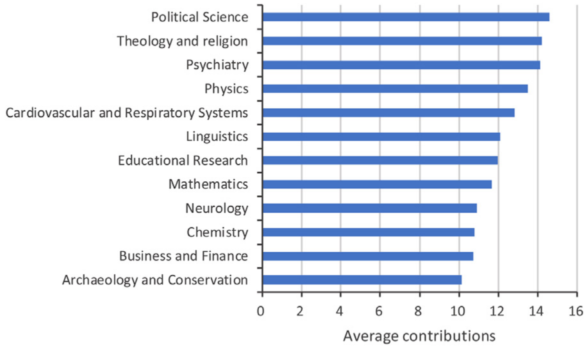

Fig. 8. Average contributions to published research of researchers in twelve subfields (three in each of four major areas of research) using MFC_2 and the NSI-based dataset with 5551 researchers (a total of 1441 researchers are included in this figure).

the 4048 scientists included in the WoS-based dataset, we can measure the proportions of their publications in NSI that are included in WoS. The shares are highest for researchers in the health sciences (84 percent) and the natural sciences and engineering (75 percent). Researchers in the social sciences (40 percent) and the humanities (32 percent) have clearly lower shares of their publications in WoS. There are also differences at the field level within each main area, e.g. between biology (86 percent) and computer science (28 percent), and between economics (65 percent) and law (30 percent). We will therefore only apply the NSI-based dataset in the following analysis.

The results shown in Fig. 7 can be compared to the results in Fig. 6 based on the same dataset and our preliminary conclusions from Section 6: While MFC_3 seems to provide a somewhat better balance in this dataset when looking at co-authorship practices only, MFC_2 provides a better balance when taking the areas of research into account. We can now conclude that MFC_2 may be preferable not only when using WoS-based data as shown in Section 6 (while disregarding biases in field representation), but also when using more complete institutional or national datasets to compare organizations with different research profiles.

Finally, we test how MFC_2 works on subfield level. Among the 84 fields in the NSI field-classification, we have selected three fields in each of the four major areas of research while ensuring that only fields with large numbers of researchers are included, and also that both higher and lower average scores according to MFC_2 are represented in the selection. The results are shown in Fig. 8.

The results in Fig. 8 on the one hand confirm that MFC_2 seems not to give any bias towards any of the major areas of research. On the other hand, the differences between the highest and lowest scores are large enough to indicate a somewhat different result from what is shown in Fig. 7: It is only at the aggregate level, at the level of major area of research, that MFC_2 (and MFC in general) can measure contributions in a balanced way.

Our explanation for this different result, is that our working hypothesis, which assumes that average contributions across fields are equal, cannot be confirmed at a real-world local level. With the four Norwegian universities as a case study, there will always be reasons why researchers in some fields (departments) contribute relatively more than others to scientific publications. One example of an explanation for such observed differences, is that some departments may have more resources from external funding than other departments at the local level.

Although our working hypothesis about equal average contributions evidently cannot be confirmed by using local real-world data, it has been useful to demonstrate that Modified Fractional Counting (MFC_2 and MFC_3) in general reduces differences in scientific contributions between co-authorship practices and fields of research whenever these contribu-

Table 8
A calculation of full counting, complete-normalized fractional counting (MFC_1) and different other values of *k* on WoS-based data for the four largest Norwegian universities[a].

	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full Counts
Oslo	7432	10734	13573	15535	17875	18053
Bergen	4395	6389	8163	9431	10991	11112
Trondheim	5719	7722	9253	10224	11301	11379
Tromsø	2369	3329	4140	4690	5330	5378
Total	19915	28174	35129	39880	45497	45922

[a] Oslo: University of Oslo; Bergen: University of Bergen; Trondheim: NTNU - Norwegian University of Science and Technology; Tromsø: UiT - The Arctic University of Norway. The same applies to Tables 9–11.

Table 9
The results in Table 8 expressed as percentages.

	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full Counts
Oslo	37.3 %	38.1 %	38.6 %	39.0 %	39.3 %	39.3 %
Bergen	22.1 %	22.7 %	23.2 %	23.6 %	24.2 %	24.2 %
Trondheim	28.7 %	27.4 %	26.3 %	25.6 %	24.8 %	24.8 %
Tromsø	11.9 %	11.8 %	11.8 %	11.8 %	11.7 %	11.7 %
Total	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %

Table 10
A calculation of full counting, complete-normalized fractional counting (MFC_1) and different other values of *k* on NSI-based data for the four largest Norwegian universities.

	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full Counts
Oslo	14566	19279	23293	26035	29265	29509
Bergen	7824	10549	12914	14568	16565	16718
Trondheim	11268	14895	17685	19456	21416	21559
Tromsø	4622	6012	7166	7940	8833	8899
Total	38270	50735	61058	67999	76079	76685

tions are measured at an aggregate level with the use of bibliographic data. It now remains to demonstrate how the counting methods work at the aggregate level with data representing real institutions.

8. Results from an application at institutional level

Our study focuses on the use of counting methods for statistics, evaluations, and funding at an aggregate level. We want to improve the comparability when organizations or countries with different research profiles are compared. Therefore, we tested how MFC works at an institutional level and across different fields of research.

The four largest Norwegian universities are represented in our data. We use the same WoS-based (4048 researchers) and NSI-based datasets (5551 researchers) as described in Data section, but the researchers' contributions are now aggregated to the institutional level. Table 8 (WoS-based) and Table 10 (NSI-based) show the numbers, while Table 9 (WoS-based) and 11 (NSI-based) show how the numbers are distributed in percentages among the four universities. All tables compare the results of full counting (deduplicated, i.e., an institution can only get one credit from the same publication, no matter how many researchers are from this same institution), fractional counting (the sum of authors' fractions per institution), and the sum of authors' MFC for different values of *k*. As an example, if a publication has been co-authored by five researchers and two of these researchers are affiliated with the university in focus, the university is credited 1 with full counting, $2/5 = 0.4$ with fractional counting, 0.63 with MFC_2, and so on.

We see in Table 9 that full and fractional counting gives different results, particularly between two of the universities (the University of Bergen vs. the Norwegian University of Science and Technology (Trondheim)). One explanation is that only one of the two universities (University of Bergen) has continuously participated in CERN, the European Organization for Nuclear Research, which very frequently publishes articles with more than 3000 authors. As much as 18 percent of the publications from University of Bergen are contributed by researchers in co-authorship groups 10, 15 and 1000. These are the groups with the highest median number of co-authors in their publications. Only 5 percent of the publications from NTNU in Trondheim fall in these high co-authorship groups.

A difference between Tables 9 and 11 is that University of Oslo has a higher share with full counting than with fractional counting in Table 9 while the opposite is the case for Table 11. This can be explained by the fact that University of Oslo has a very large Faculty of Humanities. As much as 45 percent of all publications in the humanities from the four universities are from University of Oslo. The humanities have co-authorship practices that gain larger weight with fractional counting than with full counting. This effect is more clearly seen with a NSI-based dataset, which represents the humanities more completely than with a WoS-based dataset.

Table 11
The results in Table 10 expressed as percentages.

	MFC_1	MFC_2	MFC_3	MFC_4	MFC_8	Full Counts
Oslo	38.0 %	38.0 %	38.1 %	38.3 %	38.5 %	38.5 %
Bergen	20.4 %	20.8 %	21.1 %	21.4 %	21.8 %	21.8 %
Trondheim	29.4 %	29.4 %	29.0 %	28.6 %	28.2 %	28.1 %
Tromsø	12.1 %	11.8 %	11.7 %	11.7 %	11.6 %	11.6 %
Total	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %

The main result, however, that can be derived from Tables 8–11, is that MFC_2 and MFC_3 represent an *intermediate* solution that sits between full and complete-normalized fractional counting (MFC_1) at the institutional level. We explain this with the evidence given in Sections 6 and 7: MFC_2 and MFC_3 provide a more balanced representation of different co-authorship practices. Such practices vary across fields, which is a challenge when organizations or countries with different research profiles are compared.

As noted in the introduction, harmonic counting (Hagen, 2008) is a variant of fractional counting that considers the sequence of authors. When applying harmonic counting, authors receive credit according to their rank in the byline. An author ranked in the *i*-th place receives a credit equal to:

$$\frac{\frac{1}{i}}{\left(1+\frac{1}{2}+\ldots+\frac{1}{N}\right)}$$

where N is the number of authors.

The need for harmonic counting is justified by the observation that the sequence of the authors is important in representing individual contributions in most fields of research. It is, however, difficult to imagine that one university would have more first authors than another in a large dataset like ours. Hence, our hypothesis was that using harmonic counting would not make much difference at an aggregate level. We tested this and found that percentage shares among the four major Norwegian universities are comparable (less than one percentage-point different) to Table 11 (results not shown) and that these differences tend to decrease when applying MFC_2. We conclude that the variant of fractional counting used makes little difference at the aggregate level.

9. Discussion

Given that the share of publications with cross-institutional and international co-authorship is growing, it is timely to reflect on the various counting methods for publications to gauge the extent to which they represent real-world scientific contributions. Organizations and countries are compared today with counting methods – full or fractional counting - that do not balance well between different co-authorship practices, and thereby between the different research profiles that organizations and countries may have. In addition, it can be demonstrated that the fractional counting methods do not reflect overlapping tasks and ethical responsibilities that come with teamwork and co-authorship in science.

We have developed a new counting method called modified fractional counting (MFC). The method is an intermediate counting method that sits between traditional complete-normalized fractional counting and full counting. It is specifically designed to support comparability at aggregate levels, and we have tested that it works. The method is not designed to be used at the individual level where bibliographic information cannot be used alone to understand individual contributions to scientific work. Our focus is on counting methods at the aggregate level because they are used for statistics, evaluations and strategic purposes that can make a real difference in overall policies and funding. At this level, it can be important to understand why two counting methods give quite different results in the Leiden Ranking for the two large Chinese universities that we used as examples in our introduction. We have demonstrated that neither of the two counting methods sufficiently take differences in research profiles (and thereby co-authorship practices) into account, and that modified fractional counting (MFC) may provide a more balanced representation of such differences and more consistent results.

Although we have left citation indicators out of this study, we are aware that counting methods for articles may have implications for the construction of normalized citation indicators (Fairclough & Thelwall, 2015; Perianes-Rodriguez & Ruiz-Castillo, 2015; Waltman & van Eck, 2015). We intend to include this perspective in further work on counting methods.

Author contributions

Gunnar Sivertsen: Concieved and designed the analysis; Collected the data; Contributed data or analysis tools; Performed the analysis; Wrote the paper.

Ronald Rousseau: Contributed data or analysis tools; Performed the analysis; Wrote the paper.

Lin Zhang: Concieved and designed the analysis; Contributed data or analysis tools; Performed the analysis; Wrote the paper.

Acknowledgments

Lin Zhang acknowledges support from the National Natural Science Foundation of China (Grant No. 71573085) and the Social Science Scholarship in HeNan Province (Grant 2018-YXXZ-10). Gunnar Sivertsen acknowledges the support from the FORINNPOL programme, project number 256223, at the Research Council of Norway. The authors thank Tim Engels (University of Antwerp) and anonymous reviewers for helpful remarks.

Appendix A. Mathematical foundations and implications of Modified Fractional Counting (MFC) using a sensitivity parameter

Let $S\left(x_1, x_2, \ldots, x_{A_S}\right)$ represent the array of articles of scientist S; A_S denotes the total number of articles (co-)authored by S. This array is termed the publication array of scientist S. If it is clear who wrote these articles, or when it does not matter, we simply write A instead of A_S. The contribution of scientist S, as determined by MFC with a sensitivity parameter k, where k=1,2,. . ., is defined as

$$MFC_k(S) = \sum_{j=1}^{A} \frac{1}{\sqrt[k]{x_j}} = \sum_{j=1}^{A} \left(x_j\right)^{-1/k} \tag{1}$$

Note that k acts as an index. If k = 1, MFC becomes complete-normalized fractional counting; if k extends to infinity MFC becomes full counting (consequently, we may denote full counting as MFC_∞). Recall that roots are special cases of exponentiation. Here, we only consider exponents of the form $1/k$, with k being a natural number not including zero. The k-th root of a non-negative real number x is unique and denoted as $\sqrt[k]{x}$.

If x is a natural number larger than 1, then the sequence $\left(\sqrt[k]{x}\right)_k$ is strictly decreasing with a limit of 1. Consequently, the sequence $\left(1/\sqrt[k]{x}\right)_k$ is strictly increasing, also with a limit of 1. If x=1, then all k-th roots are equal to 1.

Applying MFC using a k-th root is equivalent to giving each author of a publication with N authors a credit equal to $1/\sqrt[k]{N}$. The resulting indicator, applied to scientist S, is denoted as $\mathrm{MFC}_k(S)$. Hence, the publication as a whole receives a credit of $N/\sqrt[k]{N}$. If k decreases to 1 (complete-normalized fractional counting), this value becomes equal to 1, while if k increases to infinity it becomes N. The number k is referred to as the sensitivity parameter in MFC methods. When talking about the indicator itself, i.e., when not applied to a scientist, the simpler notation MFC_k has been used.

Before going deeper into some theoretical issues about MFC, we first illustrate its use and its consequences in three different cases. Hypothetical data are provided in Table A1 and the results with our MFC method are illustrated in Fig. A1.

Table A1
An illustrative example.

Author 1					Author 2					Author 3			
# articles	3				# articles	3				# articles	2		
# authors	3	5	6		# authors	6	50	1000		# authors	1	2	
sensitivity				sum	sensitivity				sum	sensitivity			sum
1	0.333	0.2	0.167	0.700	1	0.167	0.02	0.001	0.188	1	1	0.5	1.5
2	0.577	0.447	0.408	1.432	2	0.408	0.141	0.032	0.581	2	1	0.707	1.707
3	0.693	0.584	0.55	1.827	3	0.55	0.271	0.1	0.921	3	1	0.794	1.794
4	0.760	0.669	0.639	2.068	4	0.639	0.376	0.178	1.193	4	1	0.841	1.841
8	0.872	0.818	0.799	2.489	8	0.799	0.613	0.422	1.834	8	1	0.917	1.917
infinity	1	1	1	3	infinity	1	1	1	3	infinity	1	1	2

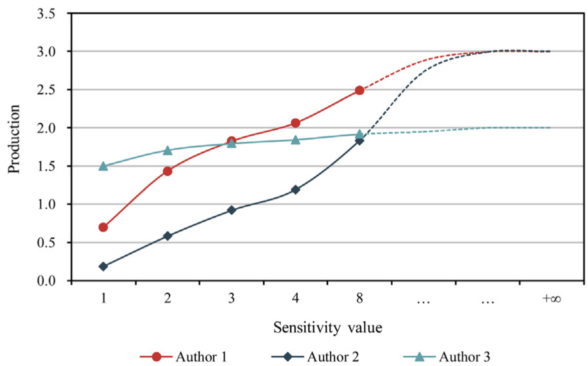

Fig. A1. The contribution of three authors depending on the sensitivity of the MFC_k indicator.

With full counting the contribution of Authors 1 and 2 would be 3, and the contribution of Author 3 would be 2. With fractional counting, Author 3 would have the highest contribution, whereas with an indicator of MFC_3 (and with sensitive parameters larger than 3, except for the infinite case) this would be Author 1. Although Author 2 has three articles and Author 3 only has two, Author 2 should be considered as having a smaller contribution than Author 3. Only for very high sensitivity values of k, MFC_k is higher for Author 2 than for Author 3.

Next, we come to some theoretical issues related to MFC.

Power functions with a fixed negative exponent (the ones we use) are monotone decreasing. Power functions with variable negative exponent applied to a fixed real number larger than one are increasing in the exponent. For example, taking 5 as the fixed positive real number, we have

$$-\frac{1}{2} < -\frac{1}{3} < -\frac{1}{4} \text{implies} (5)^{-1/2} < (5)^{-1/3} < (5)^{-1/4} \tag{2}$$

This property implies that for a given S, $MFC_k(S)$ is increasing in k. Practically, this property means that a scientist's contribution, as determined by MFC, increases monotonously from the value derived from complete-normalized fractional counting to the value derived from full counting, i.e., the number of articles. This property stays valid when summed over different scientists, as when determining the contribution of a research group, department, or country.

Now, we note some basic properties of MFC.

Property 1. If the number of a scientist's articles increases, then their MFC_k value increases for any k. This is an immediate consequence of increasing the number of terms in (1).

This basic property shows that using a sensitivity parameter results in a continuum from complete-normalized fractional counting to full counting.

Property 2. If two scientists, S_1 and S_2, each write the same number of articles with the same number of co-authors with the exception of one article by S_1, which has more co-authors than S_2, then for each finite k

$$MFC_k(S_1) < MFC_k(S_2) \tag{3}$$

This is a consequence of using monotone decreasing power functions.

Property 3. Inequality (3) also holds if S_1 and S_2 have more than one article with a different number of co-authors, given that S_1 always has at least as many co-authors as S_2.

To formulate a less obvious result, we recall the notion of majorization. Given two arrays S_1 and S_2 of equal length with the components ranked in decreasing order: $S_1\,(x_1, x_2, \ldots, x_A)$ and $S_2\,(y_1, y_2, \ldots, y_A)$. If, for each j = 1,.., A-1: $\sum_{i=1}^{j} x_i \leq \sum_{i=1}^{j} y_i$ and: $\sum_{i=1}^{A} x_i = \sum_{i=1}^{A} y_i$, then S_1 is majorized by S_2, denoted as $S_1 \preccurlyeq S_2$. Equivalently, S_2 majorizes S_1. If at least one of the inequality signs among partial sums is strict, then we have strict majorization.

Hardy, Littlewood, and Pólya, 1952, p. 89) state that, if the function f is continuous and convex and the array a with components $(aj)_{j=1,\ldots,A}$ majorizes the array b = $(bj)_{j=1,\ldots,A}$, then

$$\sum_{j=1}^{A} f(a_i) \geq \sum_{j=1}^{A} f(b_i) \tag{4}$$

When majorization is strict, then we have a strict inequality in equation (4). Applying this theorem to publication arrays yields the following proposition.

Proposition. If scientists S_1 and S_2 have the same number of articles in total with the same number of (co-)authors and if the array of S_1 strictly majorizes the array of S_2, then for each finite k, $MFC_k(S_1) > MFC_k(S_2)$.

A non-trivial application follows.

Let the array of S_1 be (10, 8, 5, 2), and let the array of S_2 be (9,9,4,3). Clearly, (10,852) majorizes (9,9,4,3). Then, for k = 2, $MSC_2\,(S_1) = \frac{1}{\sqrt{10}} + \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}} \approx 1.824 > MSC_2\,(S_2) = \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{3}} \approx 1.744$. As an illustration that this holds for every k, we do the same for k = 5: $MSC_5\,(S_1) = \frac{1}{\sqrt[5]{10}} + \frac{1}{\sqrt[5]{8}} + \frac{1}{\sqrt[5]{5}} + \frac{1}{\sqrt[5]{2}} \approx 2.886 > MSC_5\,(S_2) = \frac{1}{\sqrt[5]{9}} + \frac{1}{\sqrt[5]{9}} + \frac{1}{\sqrt[5]{4}} + \frac{1}{\sqrt[5]{3}} \approx 2.849$. Moreover, this example illustrates that MFC_k is increasing in k.

Next, we provide an application of these principles: Is MFC_k(10,863) > MFC_k(9,9,4) for every k? (Note that it makes no sense to ask for the opposite inequality as the first array has a length of 4 and the second only has a length of 3.) For any k, MFC_k(10,863) > MFC_k(10,6,3), because the first array can be derived from the second by adding one article. We chose (10,6,3), not (10,8,6) for instance, because we need an array with the same number of co-authors as (9,9,4), namely 22. While this is not possible for the current array, it is possible to consider an array with a smaller total number of co-authors, say (10,6,3). Adding 3 authors leads to (10,8,4) – the 22 co-authors needed. We know that MFC_k(10,6,3) > MFC_k(10,8,4) (which is why we chose a subarray with fewer total authors). Finally, we see that (10,8,4) majorizes (9,9,4), concluding that for each finite k, MFC_k(10,8,4) > MFC_k(9,9,4). From this we find that, for any k, MFC_k(10,863) > MFC_k(9,9,4). Note though that in many cases the MFC_k-curves for two scientists will intersect.

We conclude this appendix with an important remark. It is theoretically possible, and exceptionally so in practice, that Scientist 1 scores strictly higher than Scientist 2 with both MFC_1 (the complete-normalized fractional counting) and full

Table A2
Data and calculations for scientist S_1 (real data).

# authors	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	k= infinity
4	0.250	0.500	0.630	0.707	0.758	0.794	0.820	0.841	1
4	0.250	0.500	0.630	0.707	0.758	0.794	0.820	0.841	1
4	0.250	0.500	0.630	0.707	0.758	0.794	0.820	0.841	1
4	0.250	0.500	0.630	0.707	0.758	0.794	0.820	0.841	1
5	0.200	0.447	0.585	0.669	0.725	0.765	0.795	0.818	1
6	0.167	0.408	0.550	0.639	0.699	0.742	0.774	0.799	1
6	0.167	0.408	0.550	0.639	0.699	0.742	0.774	0.799	1
6	0.167	0.408	0.550	0.639	0.699	0.742	0.774	0.799	1
7	0.143	0.378	0.523	0.615	0.678	0.723	0.757	0.784	1
9	0.111	0.333	0.481	0.577	0.644	0.693	0.731	0.760	1
10	0.100	0.316	0.464	0.562	0.631	0.681	0.720	0.750	1
10	0.100	0.316	0.464	0.562	0.631	0.681	0.720	0.750	1
13	0.077	0.277	0.425	0.527	0.599	0.652	0.693	0.726	1
406	0.002	0.050	0.135	0.223	0.301	0.367	0.424	0.472	1
445	0.002	0.047	0.131	0.218	0.295	0.362	0.418	0.467	1
482	0.002	0.046	0.128	0.213	0.291	0.357	0.414	0.462	1
SUM	2.238	5.436	7.506	8.911	9.922	10.683	11.275	11.749	16

Table A3
Data and calculations for scientist S_2 (real data).

# authors	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	k= infinity
4	0.250	0.500	0.630	0.707	0.758	0.794	0.820	0.841	1
5	0.200	0.447	0.585	0.669	0.725	0.765	0.795	0.818	1
6	0.167	0.408	0.550	0.639	0.699	0.742	0.774	0.799	1
6	0.167	0.408	0.550	0.639	0.699	0.742	0.774	0.799	1
7	0.143	0.378	0.523	0.615	0.678	0.723	0.757	0.784	1
7	0.143	0.378	0.523	0.615	0.678	0.723	0.757	0.784	1
8	0.125	0.354	0.500	0.595	0.660	0.707	0.743	0.771	1
9	0.111	0.333	0.481	0.577	0.644	0.693	0.731	0.760	1
9	0.111	0.333	0.481	0.577	0.644	0.693	0.731	0.760	1
11	0.091	0.302	0.450	0.549	0.619	0.671	0.710	0.741	1
11	0.091	0.302	0.450	0.549	0.619	0.671	0.710	0.741	1
12	0.083	0.289	0.437	0.537	0.608	0.661	0.701	0.733	1
12	0.083	0.289	0.437	0.537	0.608	0.661	0.701	0.733	1
12	0.083	0.289	0.437	0.537	0.608	0.661	0.701	0.733	1
16	0.063	0.250	0.397	0.500	0.574	0.630	0.673	0.707	1
20	0.050	0.224	0.368	0.473	0.549	0.607	0.652	0.688	1
SUM	1.961	5.483	7.797	9.316	10.371	11.143	11.730	12.192	16

counting. However, a k exists such that Scientist 2 scores higher than Scientist 1 for MFC_k. A real-world observation of such a case can be provided with an empirical example of two Norwegian scientists S_1 and S_2 (Tables A2 and A3) with the same number of articles (16 articles) for which $\text{MFC}_1(S_1) > \text{MFC}_1(S_2)$, (2.238 versus 1.961). This might give the impression that the contribution of scientist S_1, no matter how measured, is always at least as high as that of scientist S_2. Yet this is not true when $k \geq 2$: then $\text{MFC}_k(S_1) < \text{MFC}_k(S_2)$. The largest percentage difference occurs for $k=4$, where the contribution of S_2 is about 4.5% higher than that of S_1.

References

Aagaard, K., Bloch, C., & Schneider, J. W. (2015). Impacts of performance-based research funding systems: The case of the Norwegian Publication Indicator. *Research Evaluation*, *24*(2), 106–117.

Aksnes, D. W., Schneider, J. W., & Gunnarsson, M. (2012). Ranking national research systems by citation indicators. A comparative analysis using whole and fractionalised counting methods. *Journal of Informetrics*, *6*(1), 36–43.

Bloch, C., & Schneider, J. W. (2016). Performance-based funding models and researcher behavior: An analysis of the influence of the Norwegian Publication Indicator at the individual level. *Research Evaluation*, *25*(4), 371–382.

Chai, J. C., Hua, P. H., Rousseau, R., & Wan, J. K. (2008). The adapted pure h-index. In H. Kretschmer, & F. Havemann (Eds.), *Proceedings of WIS 2008*. Berlin. http://www.collnet.de/Berlin-2008/ChaiWIS2008aph.pdf

Cronin, B. (2001). Hyperauthorship: A postmodern perversion or evidence of a structural shift in scholarly communication practices? *Journal of the American Society for Information Science and Technology*, *52*(7), 558–569.

Debackere, K., & Glänzel, W. (2004). Using a bibliometric approach to support research policy making: The case of the Flemish BOF-key. *Scientometrics*, *59*(2), 253–276.

Debackere, K., Arnold, E., Sivertsen, G., Spaapen, J., & Sturn, D. (2018). *Performance-based funding of university research*. Luxembourg: European Union.

Egghe, L., Rousseau, R., & Van Hooydonk, G. (2000). Methods for accrediting publications to authors or countries: Consequences for evaluation studies. *Journal of the American Society for Information Science*, *51*(2), 145–157.

Fairclough, R., & Thelwall, M. (2015). More precise methods for national research citation impact comparisons. *Journal of Informetrics*, *9*(4), 895–906.

Gauffriau, M., & Larsen, P. O. (2005). Counting methods are decisive for rankings based on publications and citation studies. *Scientometrics*, *64*(1), 85–93.

Gauffriau, M., Larsen, P. O., Maye, I., Roulin-Perriard, A., & von Ins, M. (2007). Publication, cooperation and productivity measures in scientific research. *Scientometrics*, *73*(2), 175–214.

Hagen, N. T. (2008). Harmonic allocation of authorship credit: Source-level correction of bibliometric bias assures accurate publication and citation analysis. *PloS One*, *3*(12) http://dx.doi.org/10.1371/journal.pone.0004021

Hardy, G., Littlewood, J. E., & Pólya, G. (1952). *Inequalities* (second edition). Cambridge (UK): Cambridge University Press.

Hirsch, J. E. (2005). An index to quantify an individual's scientific research output. *Proceedings of the National Academy of Sciences*, *102*(46), 16569–16572.

Jonkers, K., & Zacharewicz, T. (2015). *Performance based funding: A comparative assessment of their use and nature in EU Member States*. Brussels: JRC Science for Policy Report.

Katz, J. S., & Martin, B. R. (1997). What is research collaboration? *Research Policy*, *26*(-), 1–18.

Kretschmer, H., & Rousseau, R. (2001). Author inflation leads to a breakdown of Lotka's law. *Journal of the American Society for Information Science and Technology*, *52*(8), 610–614.

Martin, B. R. (1994). British science in the 1980s – Has the relative decline continued? *Scientometrics*, *29*(1), 27–56.

Marušić, A., Bošnjak, L., & Jerončić, A. (2011). A systematic review of research on the meaning, ethics and practices of authorship across scholarly disciplines. *PloS One*, *6*(9) http://dx.doi.org/10.1371/journal.pone.0023477

Moed, H. F. (2005). *Citation analysis in research evaluation*. Dordrecht: Springer.

Osório, A. (2018). On the impossibility of a perfect counting method to allocate the credits of multi-authored publications. *Scientometrics*, *116*(3), 2161–2173.

Perianes-Rodriguez, A., & Ruiz-Castillo, J. (2015). Multiplicative versus fractional counting methods for co-authored publications. The case of the 500 universities in the Leiden Ranking. *Journal of Informetrics*, *9*(4), 974–989.

Piro, F. N., Aksnes, D. W., & Rørstad, K. (2013). A macro analysis of productivity differences across fields: Challenges in the measurement of scientific publishing. *Journal of the American Society for Information Science and Technology*, *64*(2), 307–320.

Rousseau, R., Egghe, L., & Guns, R. (2018). *Becoming Metric-Wise. A bibliometric guide for researchers*. Kidlington: Elsevier-Chandos.

Sivertsen, G. (2018). The norwegian model in Norway. *Journal of Data and Information Science*, *3*(4), 2–18.

Sivertsen, G. (2016a). Publication-based funding: The Norwegian model. In M. Ochsner, S. E. Hug, & H. D. Daniel (Eds.), *Research assessment in the humanities: Towards criteria and procedures* (pp. 79–90). Zürich: Springer Open.

Sivertsen, G. (2016b). A bibliometric indicator with a balanced representation of all fields. In I. Rafols, J. Mollas-Gallart, E. Castro-Martínez, & R. Woolley (Eds.), *Proceedings of the 21st International Conference on science and technology indicators* (pp. 910–914). Valencia: Editorial Universitat Politècnica de València.

Sivertsen, G., & Larsen, B. (2012). Comprehensive bibliographic coverage of the social sciences and humanities in a citation index: An empirical analysis of the potential. *Scientometrics*, *91*(2), 567–575.

Vetenskapsrådet. (2014). *Bibliometriskt underlag för medelsfördelning*. Stockholm: The Swedish Research Council.

Waltman, L. (2016). A review of the literature on citation impact indicators. *Journal of Informetrics*, *10*(2), 365–391.

Waltman, L., & van Eck, N. J. (2015). Field-normalized citation impact indicators and the choice of an appropriate counting method. *Journal of Informetrics*, *9*(4), 872–894.